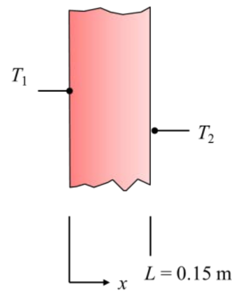
PROBLEM 1.1

KNOWN: Temperature distribution in wall of Example 1.1.

FIND: Heat fluxes and heat rates at $x = 0$ and $x = L$.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction through the wall, (2) constant thermal conductivity, (3) no internal thermal energy generation within the wall.

PROPERTIES: Thermal conductivity of wall (given): $k = 1.7$ W/m·K.

ANALYSIS: The heat flux in the wall is by conduction and is described by Fourier's law,

$$q''_x = -k\frac{dT}{dx} \tag{1}$$

Since the temperature distribution is $T(x) = a + bx$, the temperature gradient is

$$\frac{dT}{dx} = b \tag{2}$$

Hence, the heat flux is constant throughout the wall, and is

$$q''_x = -k\frac{dT}{dx} = -kb = -1.7\text{ W/m}\cdot\text{K}\times\left(-1000\text{ K/m}\right) = 1700\text{ W/m}^2 \qquad <$$

Since the cross-sectional area through which heat is conducted is constant, the heat rate is constant and is

$$q_x = q''_x \times (W \times H) = 1700\text{ W/m}^2 \times \left(1.2\text{ m} \times 0.5\text{ m}\right) = 1020\text{ W} \qquad <$$

Because the heat rate into the wall is equal to the heat rate out of the wall, steady-state conditions exist. <

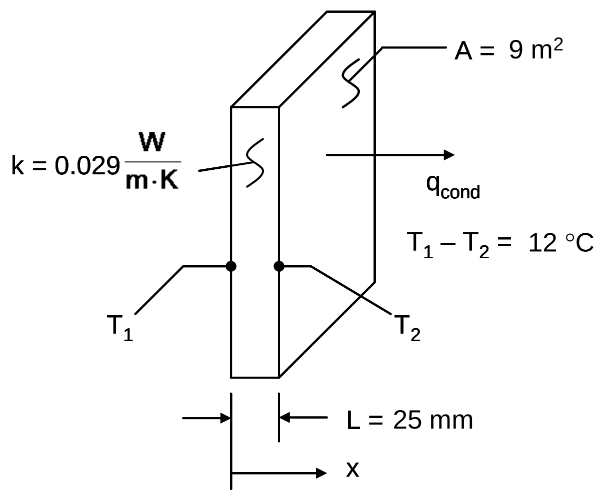
COMMENTS: (1) If the heat rates were not equal, the internal energy of the wall would be changing with time. (2) The temperatures of the wall surfaces are $T_1 = 1400$ K and $T_2 = 1250$ K.

PROBLEM 1.2

KNOWN: Thermal conductivity, thickness and temperature difference across a sheet of rigid extruded insulation.

FIND: (a) The heat flux through a 3 m × 3 m sheet of the insulation, (b) the heat rate through the sheet, and (c) the thermal conduction resistance of the sheet.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: (a) From Equation 1.2 the heat flux is

$$q''_x = -k\frac{dT}{dx} = k\frac{T_1 - T_2}{L} = 0.029\frac{W}{m \cdot K} \times \frac{12\ K}{0.025\ m} = 13.9\frac{W}{m^2} \qquad <$$

(b) The heat rate is

$$q_x = q''_x \cdot A = 13.9\frac{W}{m^2} \times 9\ m^2 = 125\ W \qquad <$$

(c) From Eq. 1.11, the thermal resistance is

$$R_{t,cond} = \Delta T / q_x = 12\ K / 125\ W = 0.096\ K/W \qquad <$$

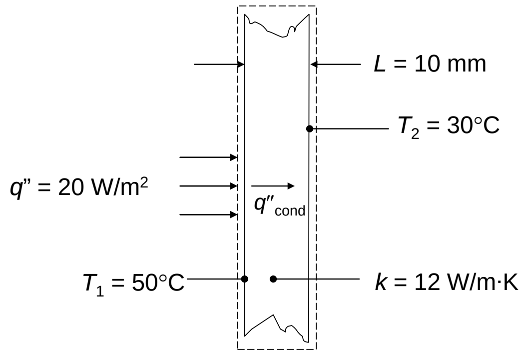
COMMENTS: (1) Be sure to keep in mind the important distinction between the heat flux (W/m^2) and the heat rate (W). (2) The direction of heat flow is from hot to cold. (3) Note that a temperature *difference* may be expressed in kelvins or degrees Celsius. (4) The conduction thermal resistance for a plane wall could equivalently be calculated from $R_{t,cond} = L/kA$.

PROBLEM 1.3

KNOWN: Thickness and thermal conductivity of a wall. Heat flux applied to one face and temperatures of both surfaces.

FIND: Whether steady-state conditions exist.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) Constant properties, (3) No internal energy generation.

ANALYSIS: Under steady-state conditions an energy balance on the control volume shown is

$$q''_{\text{in}} = q''_{\text{out}} = q''_{\text{cond}} = k(T_1 - T_2)/L = 12\ \text{W/m}\cdot\text{K}(50°\text{C} - 30°\text{C})/0.01\ \text{m} = 24{,}000\ \text{W/m}^2$$

Since the heat flux in at the left face is only 20 W/m^2, the conditions are not steady state. <

COMMENTS: If the same heat flux is maintained until steady-state conditions are reached, the steady-state temperature difference across the wall will be

$$\Delta T = q''L/k = 20\ \text{W/m}^2 \times 0.01\ \text{m}/12\ \text{W/m}\cdot\text{K} = 0.0167\ \text{K}$$

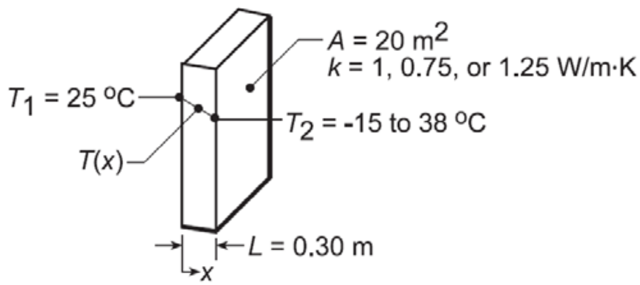
which is much smaller than the specified temperature difference of 20°C.

PROBLEM 1.4

KNOWN: Inner surface temperature and thermal conductivity of a concrete wall.

FIND: Heat loss by conduction through the wall as a function of outer surface temperatures ranging from -15 to 38°C.

SCHEMATIC:



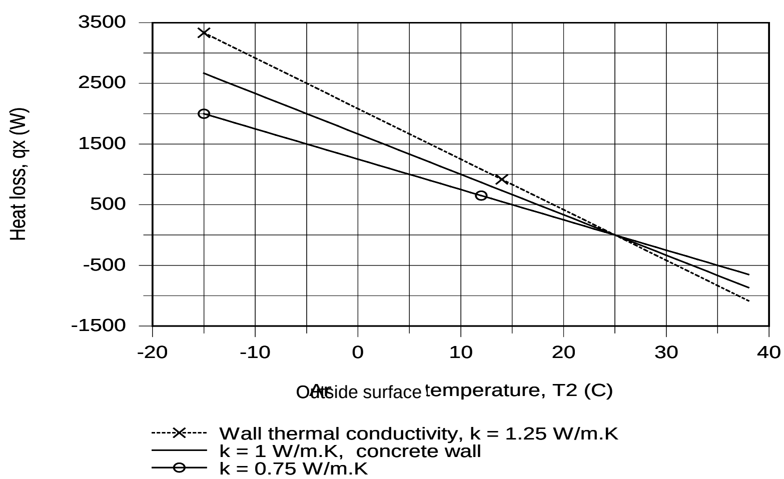
ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: From Fourier's law, if q_x'' and k are each constant it is evident that the gradient, $dT/dx = -q_x''/k$, is a constant, and hence the temperature distribution is linear. The heat flux must be constant under one-dimensional, steady-state conditions; and k is approximately constant if it depends only weakly on temperature. The heat flux and heat rate when the outside wall temperature is T_2 = -15°C are

$$q_x'' = -k\frac{dT}{dx} = k\frac{T_1 - T_2}{L} = 1\,\text{W/m}\cdot\text{K}\,\frac{25^\circ\text{C} - \left(-15^\circ\text{C}\right)}{0.30\,\text{m}} = 133.3\,\text{W/m}^2 . \qquad (1)$$

$$q_x = q_x'' \times A = 133.3\,\text{W/m}^2 \times 20\,\text{m}^2 = 2667\,\text{W} . \qquad (2) <$$

Combining Eqs. (1) and (2), the heat rate q_x can be determined for the range of outer surface temperature, $-15 \le T_2 \le 38$°C, with different wall thermal conductivities, k.



For the concrete wall, k = 1 W/m·K, the heat loss varies linearly from +2667 W to -867 W and is zero when the inside and outer surface temperatures are the same. The magnitude of the heat rate increases with increasing thermal conductivity.

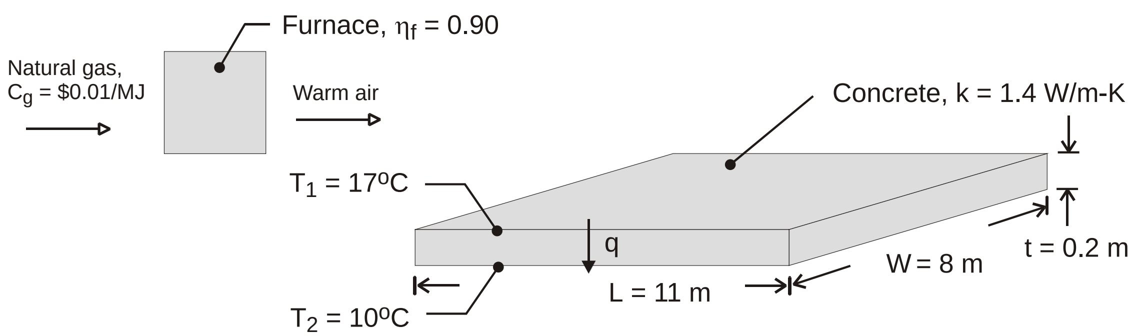
COMMENTS: Without steady-state conditions and constant k, the temperature distribution in a plane wall would not be linear.

PROBLEM 1.5

KNOWN: Dimensions, thermal conductivity and surface temperatures of a concrete slab. Efficiency of gas furnace and cost of natural gas.

FIND: Daily cost of heat loss.

SCHEMATIC:



ASSUMPTIONS: (1) Steady state, (2) One-dimensional conduction, (3) Constant properties.

ANALYSIS: The rate of heat loss by conduction through the slab is

$$q = k\,(LW)\frac{T_1 - T_2}{t} = 1.4\,\text{W/m}\cdot\text{K}\,(11\,\text{m}\times 8\,\text{m})\frac{7^\circ\text{C}}{0.20\,\text{m}} = 4312\ \text{W} \qquad <$$

The daily cost of natural gas that must be combusted to compensate for the heat loss is

$$C_d = \frac{q\,C_g}{\eta_f}(\Delta t) = \frac{4312\,\text{W}\times \$0.02/\text{MJ}}{0.9\times 10^6\,\text{J/MJ}}\left(24\,\text{h/d}\times 3600\,\text{s/h}\right) = \$8.28/\text{d} \qquad <$$

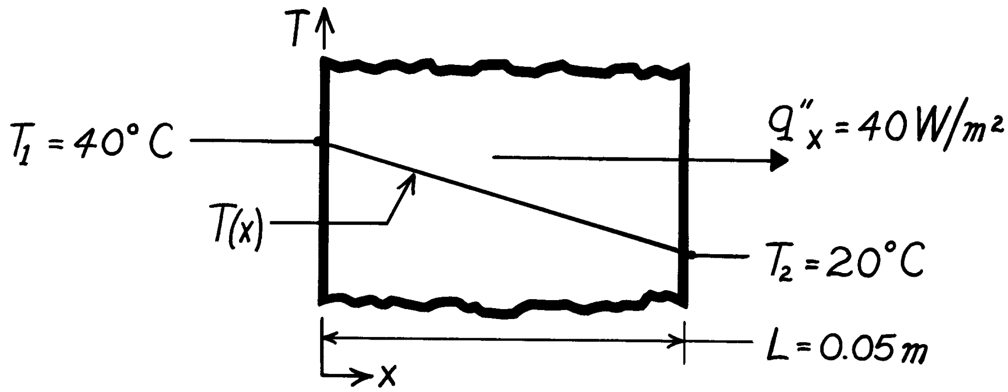
COMMENTS: The loss could be reduced by installing a floor covering with a layer of insulation between it and the concrete.

PROBLEM 1.6

KNOWN: Heat flux and surface temperatures associated with a wood slab of prescribed thickness.

FIND: Thermal conductivity, k, of the wood.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: Subject to the foregoing assumptions, the thermal conductivity may be determined from Fourier's law, Eq. 1.2. Rearranging,

$$k = q''_x \frac{L}{T_1 - T_2} = 40 \frac{W}{m^2} \frac{0.05m}{(40\text{-}20)^\circ C}$$

$$k = 0.10 \ W/m \cdot K.$$ <

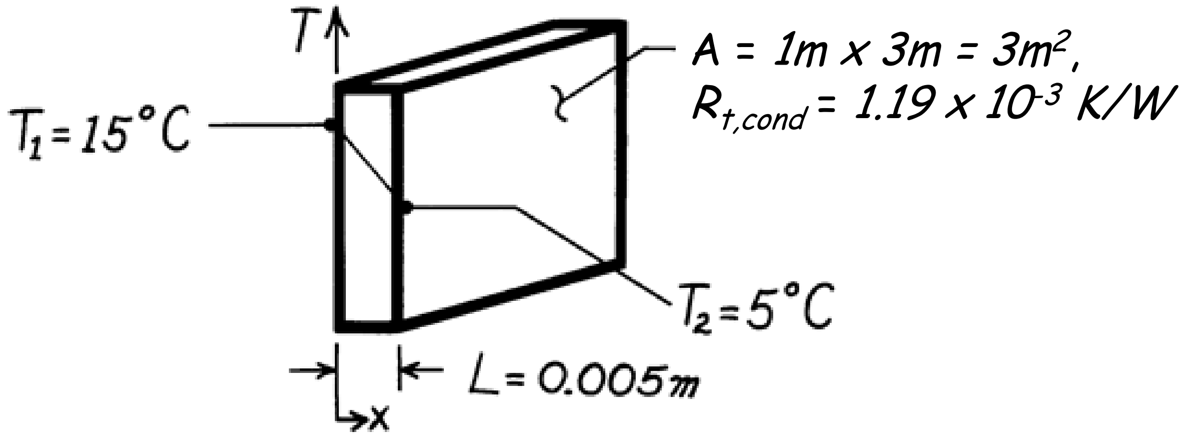
COMMENTS: Note that the °C or K temperature units may be used interchangeably when evaluating a temperature difference.

PROBLEM 1.7

KNOWN: Inner and outer surface temperatures and thermal resistance of a glass window of prescribed dimensions.

FIND: Heat loss through window. Thermal conductivity of glass.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (3) Constant properties.

ANALYSIS: From Eq. 1.11,

$$q_x = \frac{T_1 - T_2}{R_{t,cond}} = \frac{(15\text{-}5)^\circ \text{C}}{1.19 \times 10^{-3}\ \text{K/W}} = 8400\ \text{W} \qquad <$$

The thermal resistance due to conduction for a plane wall is related to the thermal conductivity and dimensions according to

$$R_{t,cond} = L/kA$$

Therefore

$$k = L/(R_{t,cond}A) = 0.005\ \text{m} / (1.19 \times 10^{-3}\ \text{K/W} \times 3\ \text{m}^2) = 1.40\ \text{W/m} \cdot \text{K} \qquad <$$

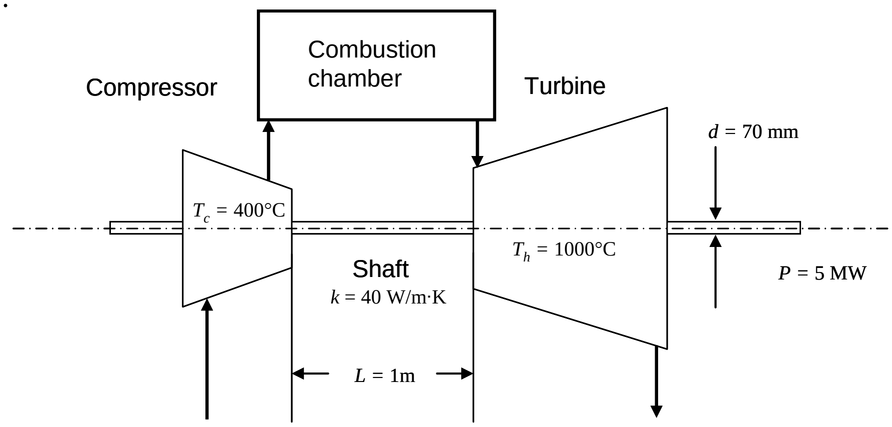
COMMENTS: The thermal conductivity value agrees with the value for glass in Table A.3.

PROBLEM 1.8

KNOWN: Net power output, average compressor and turbine temperatures, shaft dimensions and thermal conductivity.

FIND: (a) Comparison of the conduction rate through the shaft to the predicted net power output of the device, (b) Plot of the ratio of the shaft conduction heat rate to the anticipated net power output of the device over the range $0.005 \text{ m} \le L \le 1 \text{ m}$ and feasibility of a $L = 0.005$ m device.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Net power output is proportional to the volume of the gas turbine.

PROPERTIES: Shaft (given): $k = 40$ W/m·K.

ANALYSIS: (a) The conduction through the shaft may be evaluated using Fourier's law, yielding

$$q = q''A_c = \frac{k(T_h - T_c)}{L}\left(\pi d^2/4\right) = \frac{40\text{W/m}\cdot\text{K}(1000-400)^\circ\text{C}}{1\text{m}}\left(\pi(70\times10^{-3}\text{m})^2/4\right) = 92.4\text{W}$$

The ratio of the conduction heat rate to the net power output is

$$r = \frac{q}{P} = \frac{92.4\text{W}}{5\times10^6\text{W}} = 18.5\times10^{-6}$$ <

(b) The volume of the turbine is proportional to L^3. Designating $L_a = 1$ m, $d_a = 70$ mm and P_a as the shaft length, shaft diameter, and net power output, respectively, in part (a),

$$d = d_a \times (L/L_a);\ P = P_a \times (L/L_a)^3$$

and the ratio of the conduction heat rate to the net power output is

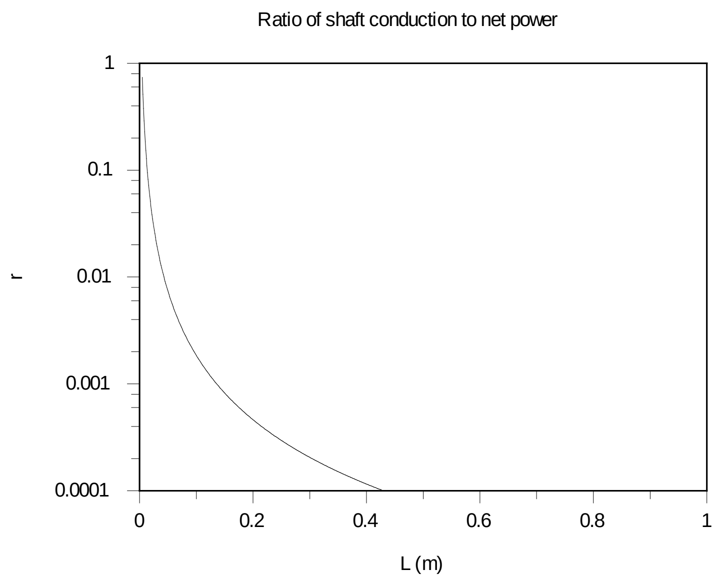
$$r = \frac{q''A_c}{P} = \frac{\frac{k(T_h - T_c)}{L}\left(\pi d^2/4\right)}{P} = \frac{\frac{k(T_h - T_c)}{L}\left(\pi\left(d_a L/L_a\right)^2/4\right)}{P_a(L/L_a)^3} = \frac{\frac{k(T_h - T_c)\pi}{4} d_a^2 L_a/P_a}{L^2}$$

$$= \frac{\frac{40\text{W/m}\cdot\text{K}(1000-400)^\circ\text{C}\pi}{4}(70\times10^{-3}\text{m})^2\times1\text{m}/5\times10^6\text{W}}{L^2} = \frac{18.5\times10^{-6}\text{m}^2}{L^2}$$

Continued…

PROBLEM 1.8 (Cont.)

The ratio of the shaft conduction to net power is shown below. At L = 0.005 m = 5 mm, the shaft conduction to net power output ratio is 0.74. The concept of the very small turbine is not feasible since it will be unlikely that the large temperature difference between the compressor and turbine can be maintained. <



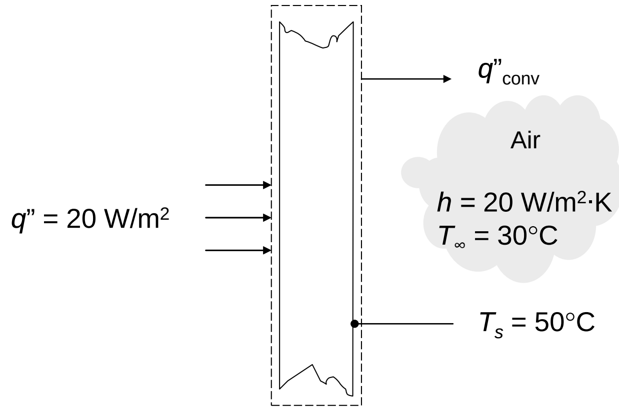
COMMENTS: (1) The thermodynamics analysis does not account for heat transfer effects and is therefore meaningful only when heat transfer can be safely ignored, as is the case for the shaft in part (a). (2) Successful miniaturization of thermal devices is often hindered by heat transfer effects that must be overcome with innovative design.

PROBLEM 1.9

KNOWN: Heat flux at one face and air temperature and convection coefficient at other face of plane wall. Temperature of surface exposed to convection.

FIND: If steady-state conditions exist. If not, whether the temperature is increasing or decreasing.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction, (2) No internal energy generation.

ANALYSIS: Conservation of energy for a control volume around the wall gives

$$\frac{dE_{st}}{dt} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g$$

$$\frac{dE_{st}}{dt} = q''_{in}A - hA(T_s - T_\infty) = \left[q''_{in} - h(T_s - T_\infty)\right]A$$

$$= \left[20\ \text{W/m}^2 - 20\ \text{W/m}^2\cdot\text{K}(50°\text{C} - 30°\text{C})\right]A = -380\ \text{W/m}^2 A$$

Since $dE_{st}/dt \neq 0$, the system is not at steady-state. <

Since $dE_{st}/dt < 0$, the stored energy is decreasing, therefore the wall temperature is decreasing. <

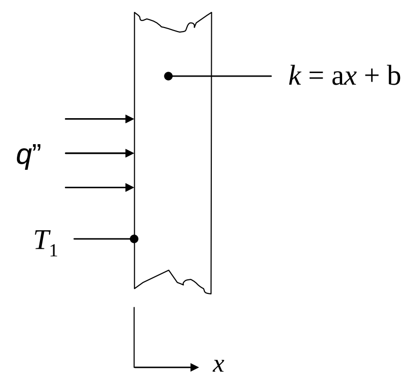
COMMENTS: When the surface temperature of the face exposed to convection cools to 31°C, $q_{in} = q_{out}$ and $dE_{st}/dt = 0$ and the wall will have reached steady-state conditions.

PROBLEM 1.10

KNOWN: Expression for variable thermal conductivity of a wall. Constant heat flux. Temperature at x = 0.

FIND: Expression for temperature gradient and temperature distribution.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction.

ANALYSIS: The heat flux is given by Fourier's law, and is known to be constant, therefore

$$q''_x = -k\frac{dT}{dx} = \text{constant}$$

Solving for the temperature gradient and substituting the expression for k yields

$$\frac{dT}{dx} = -\frac{q''_x}{k} = -\frac{q''_x}{ax+b}$$ <

This expression can be integrated to find the temperature distribution, as follows:

$$\int \frac{dT}{dx}dx = -\int \frac{q''_x}{ax+b}dx$$

Since $q''_x = \text{constant}$, we can integrate the right hand side to find

$$T = -\frac{q''_x}{a}\ln(ax+b) + c$$

where c is a constant of integration. Applying the known condition that T = T_1 at x = 0, we can solve for c.

Continued…

PROBLEM 1.10 (Cont.)

$$
\begin{aligned}
&T(x=0)=T_1 \\
&-\frac{q''_x}{a}\ln b + c = T_1 \\
&c = T_1 + \frac{q''_x}{a}\ln b
\end{aligned}
$$

Therefore, the temperature distribution is given by

$$
\begin{aligned}
T &= -\frac{q''_x}{a}\ln\left(ax+b\right) + T_1 + \frac{q''_x}{a}\ln b \qquad < \\
&= T_1 + \frac{q''_x}{a}\ln\frac{b}{ax+b} \qquad <
\end{aligned}
$$

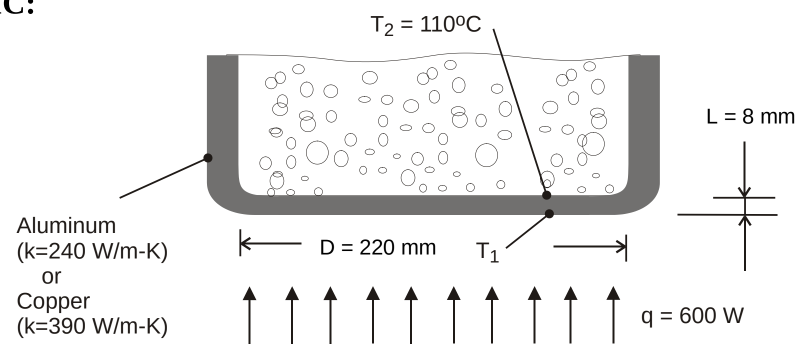
COMMENTS: Temperature distributions are not linear in many situations, such as when the thermal conductivity varies spatially or is a function of temperature. Non-linear temperature distributions may also evolve if internal energy generation occurs or non-steady conditions exist.

PROBLEM 1.11

KNOWN: Thickness, diameter and inner surface temperature of bottom of pan used to boil water. Rate of heat transfer to the pan.

FIND: Outer surface temperature of pan for an aluminum and a copper bottom.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, steady-state conduction through bottom of pan.

ANALYSIS: From Fourier's law, the rate of heat transfer by conduction through the bottom of the pan is

$$q = kA\frac{T_1 - T_2}{L}$$

Hence,

$$T_1 = T_2 + \frac{qL}{kA}$$

where $A = \pi D^2/4 = \pi(0.22\text{m})^2/4 = 0.038\ \text{m}^2$.

Aluminum: $$T_1 = 110\ ^\circ\text{C} + \frac{600\text{W}(0.008\ \text{m})}{240\ \text{W/m}\cdot\text{K}\left(0.038\ \text{m}^2\right)} = 110.5\ ^\circ\text{C}$$ $<$

Copper: $$T_1 = 110\ ^\circ\text{C} + \frac{600\text{W}(0.008\ \text{m})}{390\ \text{W/m}\cdot\text{K}\left(0.038\ \text{m}^2\right)} = 110.3\ ^\circ\text{C}$$ $<$

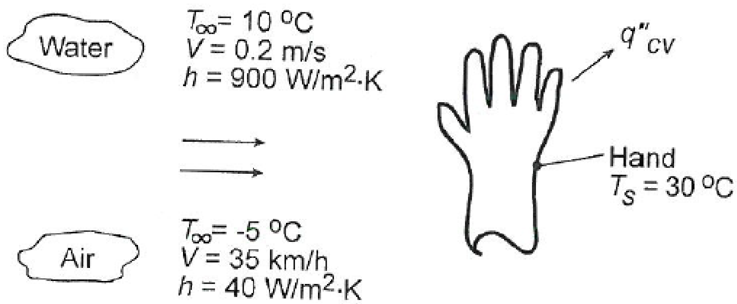
COMMENTS: Although the temperature drop across the bottom is slightly larger for aluminum (due to its smaller thermal conductivity), it is sufficiently small to be negligible for both materials. To a good approximation, the bottom may be considered *isothermal* at T ≈ 110 °C, which is a desirable feature of pots and pans.

PROBLEM 1.12

KNOWN: Hand experiencing convection heat transfer with moving air and water.

FIND: Determine which condition feels colder. Contrast these results with a heat loss of 30 W/m^2 under normal room conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Temperature is uniform over the hand's surface, (2) Convection coefficient is uniform over the hand, and (3) Negligible radiation exchange between hand and surroundings in the case of air flow.

ANALYSIS: The hand will feel colder for the condition which results in the larger heat loss. The heat loss can be determined from Newton's law of cooling, Eq. 1.3a, written as

$$q'' = h\left(T_s - T_\infty\right)$$

For the air stream:

$$q''_{air} = 40\,\mathrm{W/m^2 \cdot K}\left[30-(-8)\right]\mathrm{K} = 1{,}520\,\mathrm{W/m^2} \qquad <$$

For the water stream:

$$q''_{water} = 900\,\mathrm{W/m^2 \cdot K}\left(30-10\right)\mathrm{K} = 18{,}000\,\mathrm{W/m^2} \qquad <$$

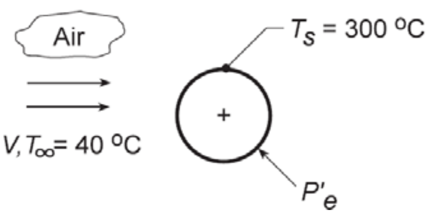
COMMENTS: The heat loss for the hand in the water stream is an order of magnitude larger than when in the air stream for the given temperature and convection coefficient conditions. In contrast, the heat loss in a normal room environment is only 30 W/m^2 which is a factor of 50 times less than the loss in the air stream. In the room environment, the hand would feel comfortable; in the air and water streams, as you probably know from experience, the hand would feel uncomfortably cold since the heat loss is excessively high.

PROBLEM 1.13

KNOWN: Power required to maintain the surface temperature of a long, 25-mm diameter cylinder with an imbedded electrical heater for different air velocities.

FIND: (a) Determine the convection coefficient for each of the air velocity conditions and display the results graphically, and (b) Assuming that the convection coefficient depends upon air velocity as $h = CV^n$, determine the parameters C and n.

SCHEMATIC:



V(m/s)	1	2	4	8	12
P'_e (W/m)	450	658	983	1507	1963
h (W/m^2·K)	22.0	32.2	48.1	73.8	96.1

ASSUMPTIONS: (1) Temperature is uniform over the cylinder surface, (2) Negligible radiation exchange between the cylinder surface and the surroundings, (3) Steady-state conditions.

ANALYSIS: (a) From an overall energy balance on the cylinder, the power dissipated by the electrical heater is transferred by convection to the air stream. Using Newton's law of cooling on a per unit length basis,

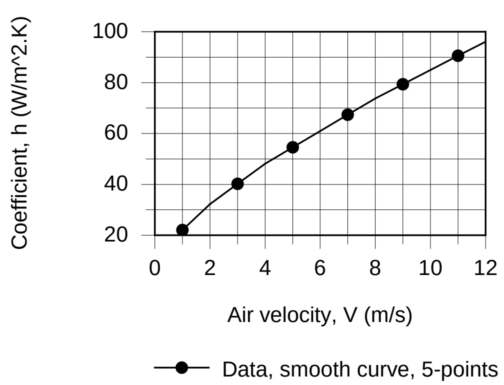
$$P'_e = h(\pi D)(T_s - T_\infty)$$

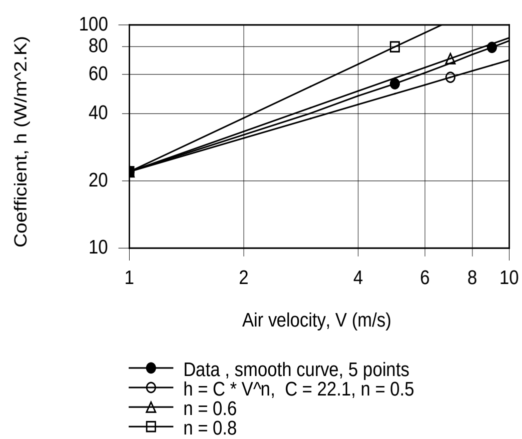
where P'_e is the electrical power dissipated per unit length of the cylinder. For the V = 1 m/s condition, using the data from the table above, find

$$h = 450\,\mathrm{W/m} \Big/ \pi \times 0.025\,\mathrm{m}\,(300-40)^\circ\mathrm{C} = 22.0\,\mathrm{W/m^2 \cdot K}$$ <

Repeating the calculations, find the convection coefficients for the remaining conditions which are tabulated above and plotted below. Note that h is not linear with respect to the air velocity.

(b) To determine the (C,n) parameters, we plotted h vs. V on log-log coordinates. Choosing C = 22.12 W/m^2·K(s/m)^n, assuring a match at V = 1, we can readily find the exponent n from the slope of the h vs. V curve. From the trials with n = 0.8, 0.6 and 0.5, we recognize that n = 0.6 is a reasonable choice. Hence, C = 22.12 and n = 0.6. <





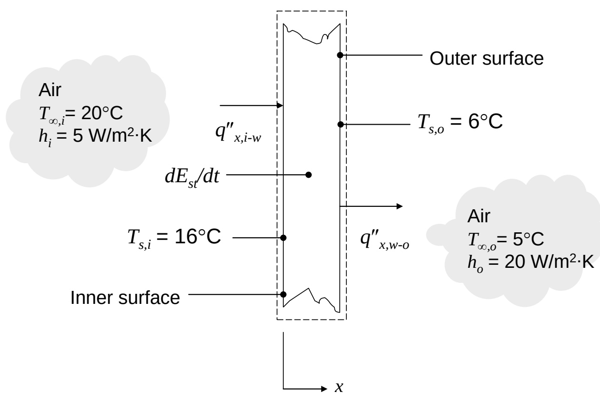
COMMENTS: Radiation may not be negligible, depending on surface emissivity.

PROBLEM 1.14

KNOWN: Inner and outer surface temperatures of a wall. Inner and outer air temperatures and convection heat transfer coefficients.

FIND: Heat flux from inner air to wall. Heat flux from wall to outer air. Heat flux from wall to inner air. Whether wall is under steady-state conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible radiation, (2) No internal energy generation.

ANALYSIS: The heat fluxes can be calculated using Newton's law of cooling. Convection from the inner air to the wall occurs in the positive x-direction:

$$q''_{x,i-w} = h_i(T_{\infty,i} - T_{s,i}) = 5\ \text{W/m}^2 \cdot \text{K} \times (20°\text{C} - 16°\text{C}) = 20\ \text{W/m}^2 \qquad <$$

Convection from the wall to the outer air also occurs in the positive x-direction:

$$q''_{x,w-o} = h_o(T_{s,o} - T_{\infty,o}) = 20\ \text{W/m}^2 \cdot \text{K} \times (6°\text{C} - 5°\text{C}) = 20\ \text{W/m}^2 \qquad <$$

From the wall to the inner air:

$$q''_{w-i} = h_i(T_{s,i} - T_{\infty,i}) = 5\ \text{W/m}^2 \cdot \text{K} \times (16°\text{C} - 20°\text{C}) = -20\ \text{W/m}^2 \qquad <$$

An energy balance on the wall gives

$$\frac{dE_{st}}{dt} = \dot{E}_{in} - \dot{E}_{out} = A(q''_{x,i-w} - q''_{x,w-o}) = 0$$

Since $dE_{st}/dt = 0$, the wall *could be* at steady-state and the *spatially-averaged* wall temperature is not changing. However, it is possible that stored energy is increasing in one part of the wall and decreasing in another, therefore we cannot tell if the wall is at steady-state or not. If we found $dE_{st}/dt \neq 0$, we would know the wall was not at steady-state. $<$

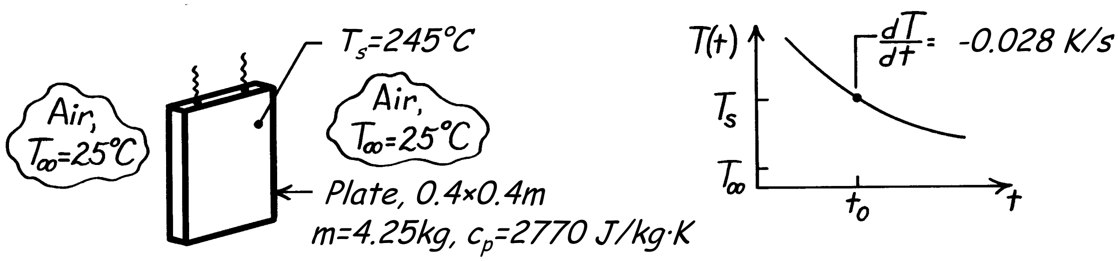
COMMENTS: The heat flux from the wall to the inner air is equal and opposite to the heat flux from the inner air to the wall.

PROBLEM 1.15

KNOWN: Hot vertical plate suspended in cool, still air. Change in plate temperature with time at the instant when the plate temperature is 245°C.

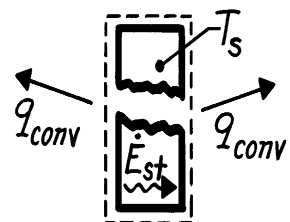
FIND: Convection heat transfer coefficient for this condition.

SCHEMATIC:



ASSUMPTIONS: (1) Plate is isothermal, (2) Negligible radiation exchange with surroundings, (3) Negligible heat lost through suspension wires.

ANALYSIS: As shown in the cooling curve above, the plate temperature decreases with time. The condition of interest is for time t_o. For a control surface about the plate, the conservation of energy requirement is



$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$$

$$-2hA_s\left(T_s - T_\infty\right) = mc_p \frac{dT}{dt}$$

where A_s is the surface area of one side of the plate. Solving for h, find

$$h = \frac{mc_p}{2A_s\left(T_s - T_\infty\right)}\left(\frac{-dT}{dt}\right)$$

$$h = \frac{4.25\ \text{kg} \times 2770\ \text{J/kg}\cdot\text{K}}{2 \times (0.4 \times 0.4)\text{m}^2 (245 - 25)\text{K}} \times 0.028\ \text{K/s} = 4.7\ \text{W/m}^2\cdot\text{K}$$ <

COMMENTS: (1) Assuming the plate is very highly polished with emissivity of 0.08, determine whether radiation exchange with the surroundings at 25°C is negligible compared to convection.

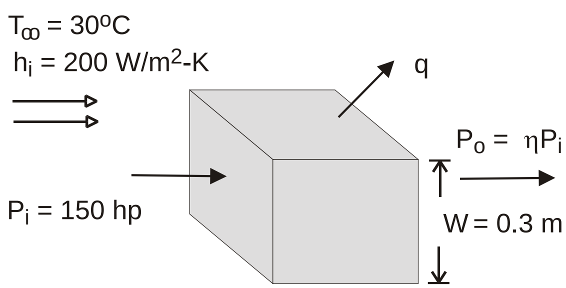
(2) We will later consider the criterion for determining whether the isothermal plate assumption is reasonable. If the thermal conductivity of the present plate were high (such as aluminum or copper), the criterion would be satisfied.

PROBLEM 1.16

KNOWN: Width, input power and efficiency of a transmission. Temperature and convection coefficient associated with air flow over the casing.

FIND: Surface temperature of casing. Thermal convection resistance.

SCHEMATIC:



ASSUMPTIONS: (1) Steady state, (2) Uniform convection coefficient and surface temperature, (3) Negligible radiation.

ANALYSIS: From Newton's law of cooling,

$$q = hA_s\left(T_s - T_\infty\right) = 6\,hW^2\left(T_s - T_\infty\right)$$

where the output power is ηP_i and the heat rate is

$$q = P_i - P_o = P_i\left(1-\eta\right) = 150\,\text{hp} \times 746\,\text{W/hp} \times 0.07 = 7833\,\text{W}$$

Hence,

$$T_s = T_\infty + \frac{q}{6\ hW^2} = 30°\text{C} + \frac{7833\ \text{W}}{6 \times 200\ \text{W/m}^2 \cdot \text{K} \times \left(0.3\,\text{m}\right)^2} = 102.5°\text{C} \qquad <$$

From Eq. 1.11, the thermal resistance due to convection is

$$R_{t,conv} = \Delta T / q_x = \left(T_s - T_\infty\right)/q_x = \left(102.5 - 30\right)\text{K} / 7833\ \text{W} = 0.00926\ \text{K/W} \qquad <$$

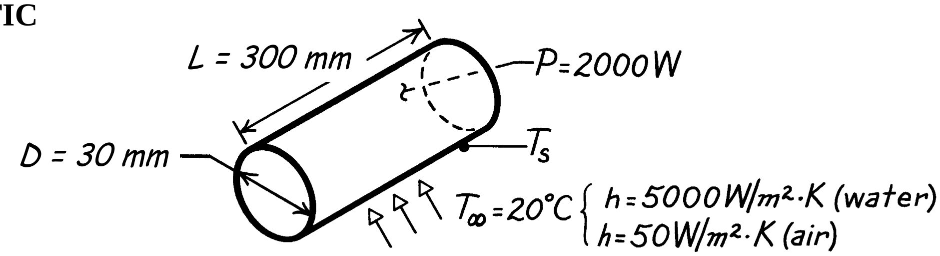
COMMENTS: (1) There will, in fact, be considerable variability of the local convection coefficient over the transmission case and the prescribed value represents an average over the surface. (2) The convection thermal resistance could equivalently be calculated from $R_{t,conv}$ = 1/hA.

PROBLEM 1.17

KNOWN: Dimensions of a cartridge heater. Heater power. Convection coefficients in air and water at a prescribed temperature.

FIND: Thermal convection resistance and heater surface temperatures in water and air.

SCHEMATIC



ASSUMPTIONS: (1) Steady-state conditions, (2) All of the electrical power is transferred to the fluid by convection, (3) Negligible heat transfer from ends.

ANALYSIS: With P = q_{conv}, Newton's law of cooling yields

$$P=hA\left(T_S-T_\infty\right)=h\pi DL\left(T_S-T_\infty\right)$$
$$T_S=T_\infty+\frac{P}{h\pi DL}.$$

From Eq. 1.11, the thermal resistance due to convection is given by

$$R_{t,conv}=\Delta T/q_x=\left(T_s-T_\infty\right)/P=1/h\pi DL$$

In water,

$$T_s=20°C+\frac{2000\text{ W}}{5000\text{ W}/\text{m}^2\cdot\text{K}\times\pi\times0.03\text{ m}\times0.3\text{ m}}=34.2°C \quad <$$

$$R_{t,conv}=1/h\pi DL=1/(5000\text{ W/m}^2\cdot\text{K}\times\pi\times0.03\text{ m}\times0.3\text{ m})=0.00707\text{ K/W} \quad <$$

In air,

$$T_s=20°C+\frac{2000\text{ W}}{50\text{ W}/\text{m}^2\cdot\text{K}\times\pi\times0.03\text{ m}\times0.3\text{ m}}=1435°C \quad <$$

$$R_{t,conv}=1/h\pi DL=1/(50\text{ W/m}^2\cdot\text{K}\times\pi\times0.03\text{ m}\times0.3\text{ m})=0.707\text{ K/W} \quad <$$

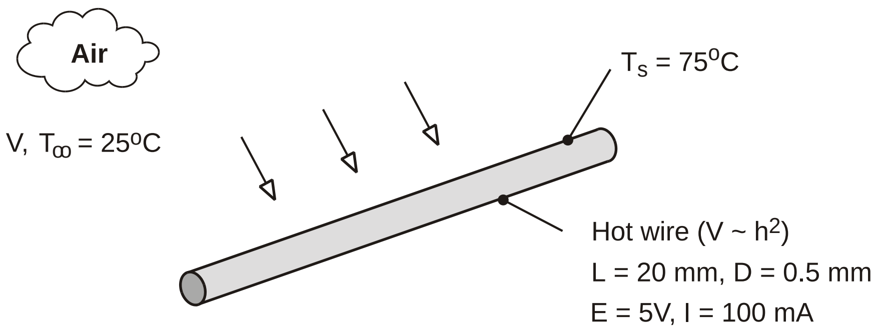
COMMENTS: (1) Air is much less effective than water as a heat transfer fluid. Hence, the cartridge temperature is much higher in air, so high, in fact, that the cartridge would melt. (2) In air, the high cartridge temperature would render radiation significant. (3) Larger thermal resistance corresponds to less effective heat transfer.

PROBLEM 1.18

KNOWN: Length, diameter and calibration of a hot wire anemometer. Temperature of air stream. Current, voltage drop and surface temperature of wire for a particular application.

FIND: Air velocity

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer from the wire by natural convection or radiation.

ANALYSIS: If all of the electric energy is transferred by convection to the air, the following equality must be satisfied

$$P_{elec} = EI = hA\left(T_s - T_\infty\right)$$

where $A = \pi DL = \pi\left(0.0005\text{m} \times 0.02\text{m}\right) = 3.14 \times 10^{-5}\text{m}^2$.

Hence,

$$h = \frac{EI}{A\left(T_s - T_\infty\right)} = \frac{5\text{V} \times 0.1\text{A}}{3.14 \times 10^{-5}\text{m}^2\left(50\ ^\circ\text{C}\right)} = 318\ \text{W/m}^2 \cdot \text{K}$$

$$V = 6.25 \times 10^{-5} h^2 = 6.25 \times 10^{-5}\left(318\ \text{W/m}^2 \cdot \text{K}\right)^2 = 6.3\ \text{m/s} \qquad <$$

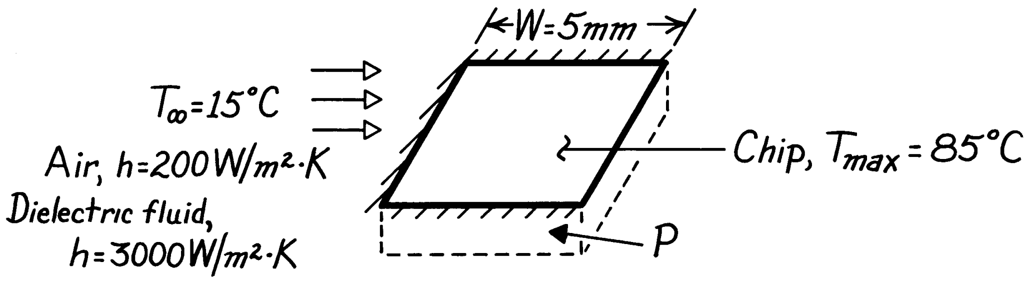
COMMENTS: The convection coefficient is sufficiently large to render buoyancy (natural convection) and radiation effects negligible.

PROBLEM 1.19

KNOWN: Chip width and maximum allowable temperature. Coolant conditions.

FIND: Maximum allowable chip power for air and liquid coolants.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible heat transfer from sides and bottom, (3) Chip is at a uniform temperature (isothermal), (4) Negligible heat transfer by radiation in air.

ANALYSIS: All of the electrical power dissipated in the chip is transferred by convection to the coolant. Hence,

$$P = q$$

and from Newton's law of cooling,

$$P = hA(T - T_\infty) = h\,W^2(T - T_\infty).$$

In *air*,

$$P_{max} = 200\ W/m^2\cdot K(0.005\ m)^2(85 - 15)\ °C = 0.35\ W. \quad <$$

In the *dielectric liquid*

$$P_{max} = 3000\ W/m^2\cdot K(0.005\ m)^2(85\text{-}15)\ °C = 5.25\ W. \quad <$$

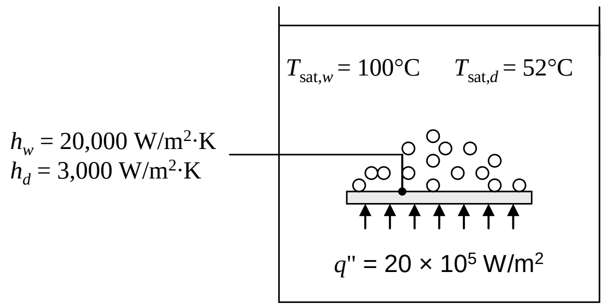
COMMENTS: Relative to liquids, air is a poor heat transfer fluid. Hence, in air the chip can dissipate far less energy than in the dielectric liquid.

PROBLEM 1.20

KNOWN: Heat flux and convection heat transfer coefficient for boiling water. Saturation temperature and convection heat transfer coefficient for boiling dielectric fluid.

FIND: Upper surface temperature of plate when water is boiling. Whether plan for minimizing surface temperature by using dielectric fluid will work.

SCHEMATIC:



ASSUMPTIONS: Steady-state conditions.

PROPERTIES: $T_{sat,w}$ = 100°C at p = 1 atm.

ANALYSIS: According to the problem statement, Newton's law of cooling can be expressed for a boiling process as

$$q'' = h(T_s - T_{sat})$$

Thus,

$$T_s = T_{sat} + q''/h$$

When the fluid is water,

$$T_{s,w} = T_{sat,w} + q''/h_w = 100°\text{C} + \frac{20 \times 10^5 \text{ W/m}^2}{20 \times 10^3 \text{ W/m}^2 \cdot \text{K}} = 200°\text{C}$$

When the dielectric fluid is used,

$$T_{s,d} = T_{sat,d} + q''/h_d = 52°\text{C} + \frac{20 \times 10^5 \text{ W/m}^2}{3 \times 10^3 \text{ W/m}^2 \cdot \text{K}} = 719°\text{C}$$

Thus, the technician's proposed approach will not reduce the surface temperature. <

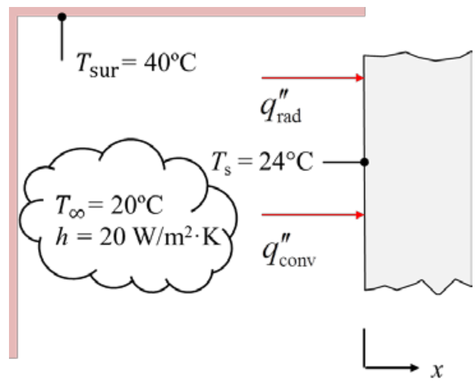
COMMENTS: (1) Even though the dielectric fluid has a lower saturation temperature, this is more than offset by the lower heat transfer coefficient associated with the dielectric fluid. The surface temperature with the dielectric coolant exceeds the melting temperature of many metals such as aluminum and aluminum alloys. (2) Dielectric fluids are, however, employed in applications such as *immersion cooling* of electronic components, where an electrically-conducting fluid such as water could not be used.

PROBLEM 1.21

KNOWN: Ambient, surface, and surroundings temperatures, convection heat transfer coefficient, and absorptivity of a plane wall.

FIND: Convective and radiative heat fluxes to the wall at $x = 0$.

SCHEMATIC:



ASSUMPTIONS: (1) Exposed wall surface is gray ($\alpha = \varepsilon$), (2) large surroundings.

ANALYSIS: The convection heat flux to the wall is described by Newton's law of cooling,

$$q''_{\text{conv}} = h(T_\infty - T_1) = 20\text{W/m}^2 \cdot \text{K} \times (20°\text{C} - 24°\text{C}) = -80\ \text{W/m}^2 \qquad <$$

The negative sign indicates that the convection heat transfer is from the wall to the ambient.

The net radiation heat flux to the wall is determined from

$$q''_{\text{rad}} = \varepsilon\sigma\left(T_{\text{sur}}^4 - T_s^4\right) = 0.78 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4 \times \left(\left(40+273\right)^4 - \left(24+273\right)^4\right)\text{K}^4 = 80\text{W/m}^2 <$$

The net radiation heat flux is to the wall from the surroundings.

Since the radiation and convection heat fluxes are equal and opposite, the net heat flux to the wall is zero.

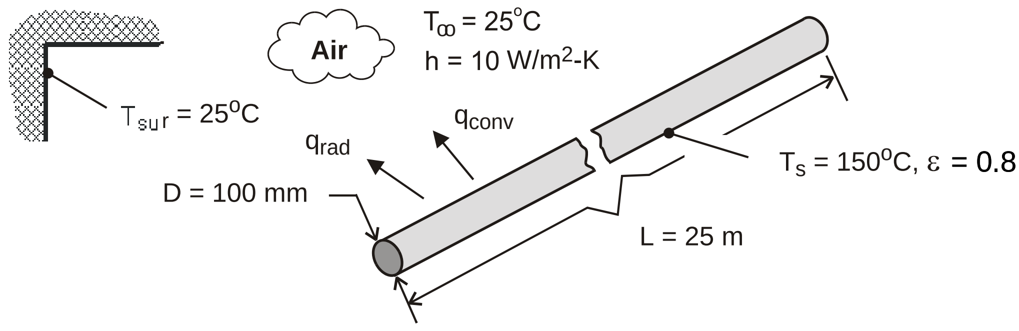
COMMENTS: (1) If the wall is constructed of a thermally-insulating material, its thermal conductivity will be small, and the conduction heat flux inside the wall will also be small. This situation leads to the requirement that the *sum* of the convective and net radiative fluxes at $x = 0$ be small, such as the case here. (2) Note the importance of converting the temperatures to kelvins when solving for the radiation heat flux.

PROBLEM 1.22

KNOWN: Length, diameter, surface temperature and emissivity of steam line. Temperature and convection coefficient associated with ambient air. Efficiency and fuel cost for gas fired furnace.

FIND: (a) Rate of heat loss, (b) Annual cost of heat loss.

SCHEMATIC:



ASSUMPTIONS: (1) Steam line operates continuously throughout year, (2) Net radiation transfer is between small surface (steam line) and large enclosure (plant walls).

ANALYSIS: (a) From Eqs. (1.3a) and (1.7), the heat loss is

$$q = q_{conv} + q_{rad} = A\left[h\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]$$

where $A = \pi DL = \pi\left(0.1\text{m} \times 25\text{m}\right) = 7.85\text{m}^2$.

Hence,

$$q = 7.85\text{m}^2\left[10\ \text{W/m}^2\cdot\text{K}\left(150-25\right)\text{K} + 0.8\times5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(423^4 - 298^4\right)\text{K}^4\right]$$

$$q = 7.85\text{m}^2\left(1,250 + 1,095\right)\text{W/m}^2 = \left(9813 + 8592\right)\text{W} = 18,405\ \text{W}$$ <

(b) The annual energy loss is

$$E = qt = 18,405\ \text{W}\times3600\ \text{s/h}\times24\text{h/d}\times365\ \text{d/y} = 5.80\times10^{11}\ \text{J}$$

With a furnace energy consumption of $E_f = E/\eta_f = 6.45\times10^{11}$ J, the annual cost of the loss is

$$C = C_g E_f = 0.02\ \$/\text{MJ}\times6.45\times10^5\text{MJ} = \$12,900$$ <

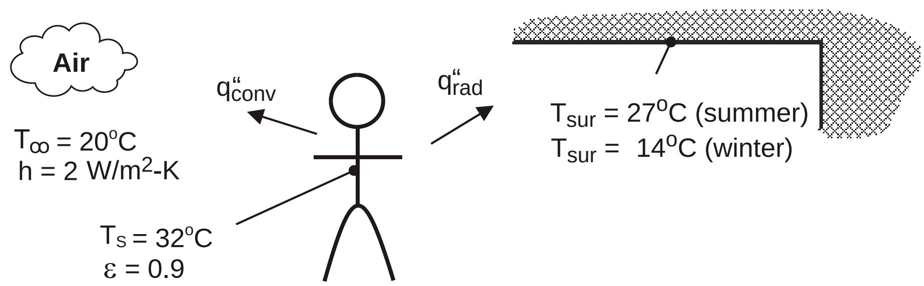
COMMENTS: The heat loss and related costs are unacceptable and should be reduced by insulating the steam line.

PROBLEM 1.23

KNOWN: Air and wall temperatures of a room. Surface temperature, convection coefficient and emissivity of a person in the room.

FIND: Basis for difference in comfort level between summer and winter. Ratio of thermal convection resistance to thermal radiation resistance in summer and winter.

SCHEMATIC:



ASSUMPTIONS: (1) Person may be approximated as a small object in a large enclosure.

ANALYSIS: Thermal comfort is linked to heat loss from the human body, and a *chilled* feeling is associated with excessive heat loss. Because the temperature of the room air is fixed, the different summer and winter comfort levels cannot be attributed to convection heat transfer from the body. In both cases, the convection heat flux is

Summer and Winter: $q''_{conv} = h\left(T_s - T_\infty\right) = 2\ \text{W/m}^2 \cdot \text{K} \times 12\ ^\circ\text{C} = 24\ \text{W/m}^2$

However, the heat flux due to radiation will differ, with values of

Summer: $q''_{rad} = \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) = 0.9 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4\left(305^4 - 300^4\right)\text{K}^4 = 28.3\ \text{W/m}^2$

Winter: $q''_{rad} = \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) = 0.9 \times 5.67 \times 10^{-8}\,\text{W/m}^2 \cdot \text{K}^4\left(305^4 - 287^4\right)\text{K}^4 = 95.4\ \text{W/m}^2$

There is a significant difference between winter and summer radiation fluxes, and the chilled condition is attributable to the effect of the colder walls on radiation. <

From Eq. 1.11, the thermal resistance due to convection is

$$R_{t,conv} = \Delta T / q_{conv} = \left(T_s - T_\infty\right) / q''_{conv}A$$

and the thermal resistance due to radiation is

$$R_{t,rad} = \Delta T / q_{rad} = \left(T_s - T_{sur}\right) / q''_{rad}A$$

Continued…

Thus the ratio of resistances is

$$\frac{R_{t,conv}}{R_{t,rad}} = \frac{\left(T_s - T_\infty\right)/q''_{conv}}{\left(T_s - T_{sur}\right)/q''_{rad}}$$

Summer:
$$\frac{R_{t,conv}}{R_{t,rad}} = \frac{(32-20)\ \text{K}/24\ \text{W/m}^2}{(32-27)\ \text{K}/28.3\ \text{W/m}^2} = 2.83 \qquad <$$

Winter:
$$\frac{R_{t,conv}}{R_{t,rad}} = \frac{(32-20)\ \text{K}/24\ \text{W/m}^2}{(32-14)\ \text{K}/95.4\ \text{W/m}^2} = 2.65 \qquad <$$

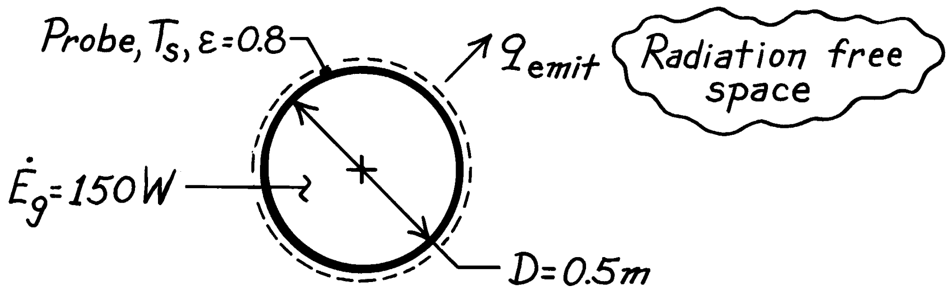
COMMENTS: (1) For a representative surface area of A = 1.5 m^2, the heat losses are q_{conv} = 36 W, $q_{rad(summer)}$ = 42.5 W and $q_{rad(winter)}$ = 143.1 W. The winter time radiation loss is significant and if maintained over a 24 h period would amount to 2,950 kcal. (2) The convection resistance is larger than the radiation resistance but they are the same order of magnitude. There isn’t much difference in the resistances between summer and winter conditions; the main difference is the larger *temperature difference* through which radiation occurs in the winter as compared to summer.

PROBLEM 1.24

KNOWN: Diameter and emissivity of spherical interplanetary probe. Power dissipation within probe.

FIND: Probe surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation incident on the probe.

ANALYSIS: Conservation of energy dictates a balance between energy generation within the probe and radiation emission from the probe surface. Hence, at any instant

$$-\dot{E}_{out} + \dot{E}_g = 0$$

$$\varepsilon A_s \sigma T_s^4 = \dot{E}_g$$

$$T_s = \left(\frac{\dot{E}_g}{\varepsilon \pi D^2 \sigma} \right)^{1/4}$$

$$T_s = \left(\frac{150\text{W}}{0.8\pi (0.5\text{ m})^2\, 5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4} \right)^{1/4}$$

$$T_s = 254.7\text{ K.}$$ <

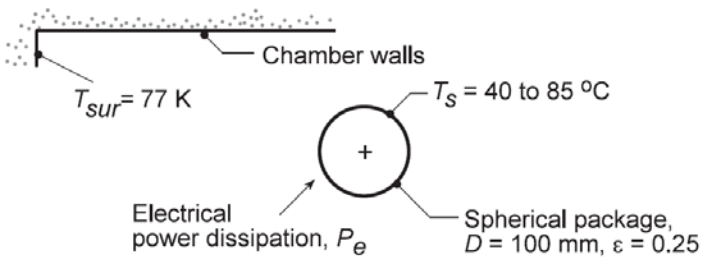
COMMENTS: Incident radiation, as, for example, from the sun, would increase the surface temperature.

PROBLEM 1.25

KNOWN: Spherical shaped instrumentation package with prescribed surface emissivity within a large space-simulation chamber having walls at 77 K.

FIND: Acceptable power dissipation for operating the package surface temperature in the range T_s = 40 to 85°C. Show graphically the effect of emissivity variations for 0.2 and 0.3.

SCHEMATIC:



ASSUMPTIONS: (1) Uniform surface temperature, (2) Chamber walls are large compared to the spherical package, and (3) Steady-state conditions.

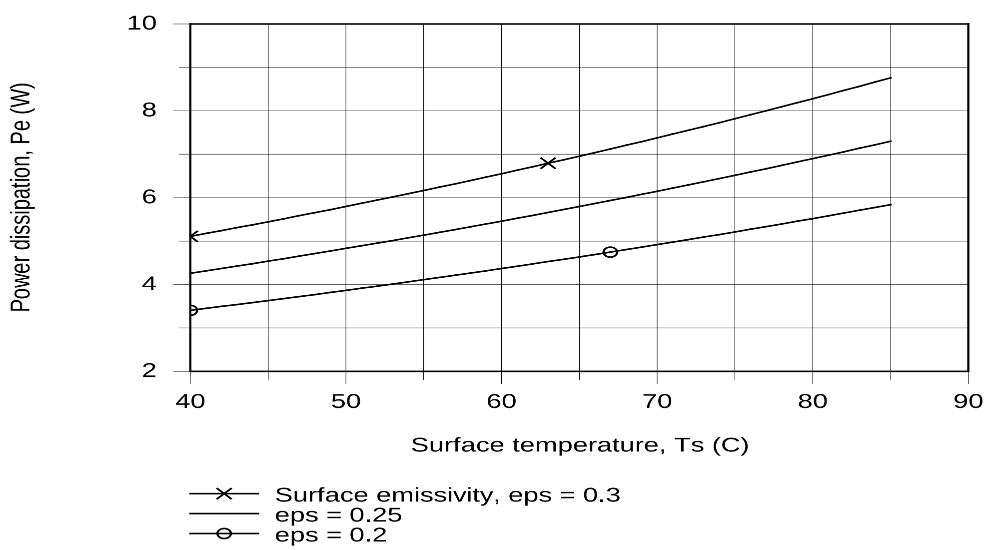
ANALYSIS: From an overall energy balance on the package, the internal power dissipation P_e will be transferred by radiation exchange between the package and the chamber walls. From Eq. 1.7,

$$q_{rad} = P_e = \varepsilon A_s \sigma \left(T_s^4 - T_{sur}^4 \right)$$

For the condition when T_s = 40°C, with $A_s = \pi D^2$ the power dissipation will be

$$P_e = 0.25\left(\pi \times 0.10^2 \text{ m}^2\right) \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 \times \left[(40 + 273)^4 - 77^4 \right] \text{K}^4 = 4.3 \text{ W} \qquad <$$

Repeating this calculation for the range $40 \le T_s \le 85$°C, we can obtain the power dissipation as a function of surface temperature for the ε = 0.25 condition. Similarly, with 0.2 or 0.3, the family of curves shown below has been obtained.



COMMENTS: (1) As expected, the internal power dissipation increases with increasing emissivity and surface temperature. Because the radiation rate equation is non-linear with respect to temperature, the power dissipation will likewise not be linear with surface temperature.

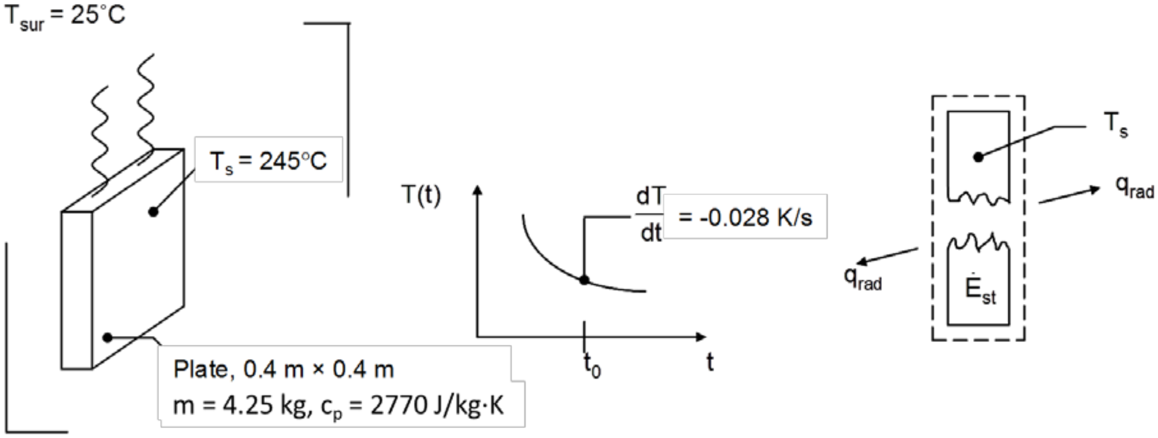
(2) What is the maximum power dissipation that is possible if the surface temperature is not to exceed 85°C? What kind of a coating should be applied to the instrument package in order to approach this limiting condition?

PROBLEM 1.26

KNOWN: Hot plate suspended in vacuum and surroundings temperature. Mass, specific heat, area and time rate of change of plate temperature.

FIND: (a) The emissivity of the plate, and (b) The rate at which radiation is emitted from the plate.

SCHEMATIC:



ASSUMPTIONS: (1) Plate is isothermal and at uniform temperature, (2) Large surroundings, (3) Negligible heat loss through suspension wires.

ANALYSIS: For a control volume about the plate, the conservation of energy requirement is

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st} \tag{1}$$

where $\dot{E}_{st} = mc_p \frac{dT}{dt}$ (2)

and for large surroundings $\dot{E}_{in} - \dot{E}_{out} = A\varepsilon\sigma(T_{sur}^4 - T_s^4)$ (3)

Combining Eqns. (1) through (3) yields

$$\varepsilon = \frac{mc_p}{A\sigma} \frac{\frac{dT}{dt}}{(T_{sur}^4 - T_s^4)}$$

Noting that T_{sur} = 25˚C + 273 K = 298 K and T_s = 245˚C + 273 K = 518 K, we find

$$\varepsilon = \frac{4.25 \text{ kg} \times 2770 \frac{\text{J}}{\text{kg}\cdot\text{K}} \times (-0.028 \frac{\text{K}}{\text{s}})}{2 \times 0.4 \text{ m} \times 0.4 \text{ m} \times 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2\cdot\text{K}^4}(298^4 - 518^4)\text{ K}^4} = 0.28$$ <

The rate at which radiation is emitted from the plate is

$$q_{rad,e} = \varepsilon A\sigma T_s^4 = 0.28 \times 2 \times 0.4 \text{ m} \times 0.4 \text{ m} \times 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2\cdot\text{K}^4} \times (518 \text{ K})^4 = 370 \text{ W}$$ <

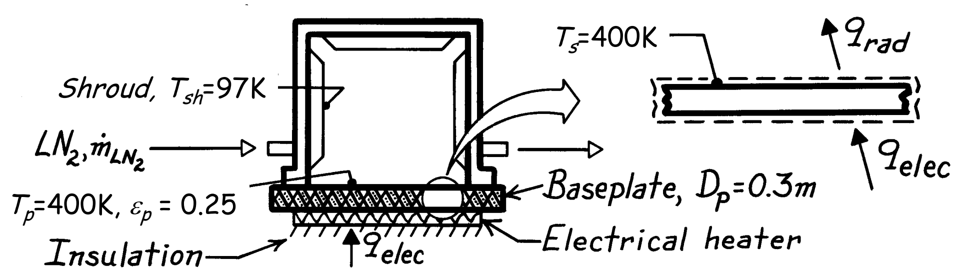
COMMENTS: Note the importance of using kelvins when working with radiation heat transfer.

PROBLEM 1.27

KNOWN: Vacuum enclosure maintained at 97 K by liquid nitrogen shroud while baseplate is maintained at 400 K by an electrical heater.

FIND: (a) Electrical power required to maintain baseplate, (b) Liquid nitrogen consumption rate, (c) Effect on consumption rate if aluminum foil (ε_p = 0.09) is bonded to baseplate surface.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) No heat losses from backside of heater or sides of plate, (3) Vacuum enclosure large compared to baseplate, (4) Enclosure is evacuated with negligible convection, (5) Liquid nitrogen (LN_2) is heated only by heat transfer to the shroud, and (6) Foil is intimately bonded to baseplate.

PROPERTIES: Heat of vaporization of liquid nitrogen (given): 125 kJ/kg.

ANALYSIS: (a) From an energy balance on the baseplate,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_{elec} - q_{rad} = 0$$

and using Eq. 1.7 for radiative exchange between the baseplate and shroud,

$$q_{elec} = \varepsilon_p A_p \sigma \left(T_p^4 - T_{sh}^4\right).$$

Substituting numerical values, with $A_p = \left(\pi D_p^2/4\right)$, find

$$q_{elec} = 0.25\left(\pi(0.3\text{ m})^2/4\right)5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\left(400^4 - 97^4\right)\text{K}^4 = 25.6\text{ W}. \qquad <$$

(b) From an energy balance on the enclosure, radiative transfer heats the liquid nitrogen stream causing evaporation,

$$\dot{E}_{in} - \dot{E}_{out} = 0 \qquad q_{rad} - \dot{m}_{LN2}h_{fg} = 0$$

where $\dot{m}_{LN2}$ is the liquid nitrogen consumption rate. Hence,

$$\dot{m}_{LN2} = q_{rad}/h_{fg} = 25.6\text{ W}/125\text{ kJ/kg} = 2.05\times10^{-4}\text{ kg/s} = 0.736\text{ kg/h}. \qquad <$$

(c) If aluminum foil (ε_p = 0.09) were bonded to the upper surface of the baseplate,

$$q_{rad,foil} = q_{rad}\left(\varepsilon_f/\varepsilon_p\right) = 25.6\text{ W}(0.09/0.25) = 9.2\text{ W}$$

and the liquid nitrogen consumption rate would be reduced by

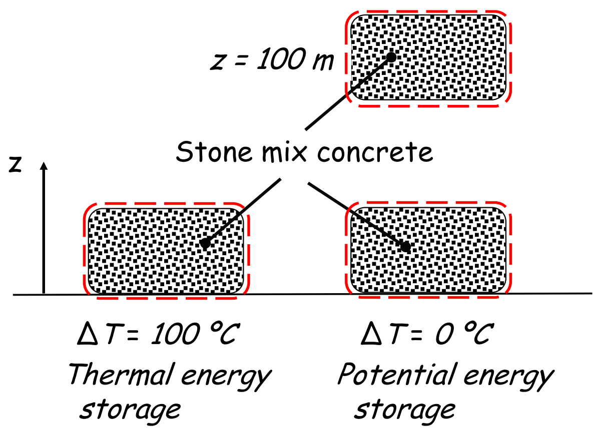
$$(0.25 - 0.09)/0.25 = 64\%\text{ to }0.265\text{ kg/h}. \qquad <$$

PROBLEM 1.28

KNOWN: Storage medium, minimum and maximum temperatures for thermal energy storage, vertical elevation change for potential energy storage.

FIND: Ratio of sensible energy storage capacity to potential energy storage capacity.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Uniform minimum and maximum temperatures.

PROPERTIES: Table A.3, stone mix concrete (300 K): $c_p = c = 880$ J/kg·K.

ANALYSIS: The change in sensible thermal energy storage is due to the temperature change, thus

$$\dot{E}_{\mathrm{st},t} = \frac{dU_t}{dt} = mc\frac{dT}{dt} \tag{1}$$

Integrating Equation 1 over the time period between the minimum and maximum temperatures associated with the thermal energy storage yields

$$\Delta E_{\mathrm{st},t} = mc\left(T_{\max} - T_{\min}\right) = mc\Delta T \tag{2}$$

The potential energy storage capacity is

$$\Delta E_{\mathrm{st,PE}} = mgz \tag{3}$$

Combining Equations 2 and 3 yields

$$R = \frac{c\Delta T}{gz}$$ <

Continued …

PROBLEM 1.28 (Cont.)

For stone mix concrete with ΔT = 100°C and z = 100 m,

$$R = \frac{880\ \text{J/kg}\cdot\text{K} \times 100\ \text{K}}{9.8\ \text{m/s}^2 \times 100\ \text{m}} = 89.8$$

Since $R >> 1$, thermal energy storage is more effective for the parameters of this problem. <

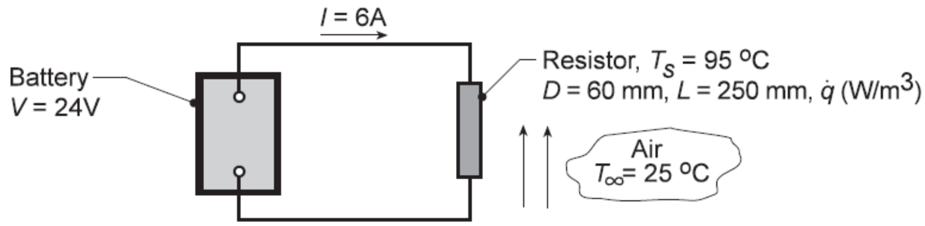
COMMENTS: (1) Note that ΔT = 100°C = 100 K. (2) The ratio, R, is dimensionless. We shall utilize dimensionless parameters frequently in later chapters. (3) Thermal energy storage can be used in conjunction with solar thermal energy generation. In general, storage of energy in a mechanical form is not as effective as in thermal or chemical forms.

PROBLEM 1.29

KNOWN: Resistor connected to a battery operating at a prescribed temperature in air.

FIND: (a) Considering the resistor as the system, determine corresponding values of $\dot{E}_{in}(W)$, $\dot{E}_g(W)$, $\dot{E}_{out}(W)$ and $\dot{E}_{st}(W)$. If a control surface is placed about the entire system, determine the values of $\dot{E}_{in}$, $\dot{E}_g$, $\dot{E}_{out}$, and $\dot{E}_{st}$. (b) Determine the volumetric heat generation rate within the resistor, $\dot{q}$ (W/m^3), (c) Neglecting radiation from the resistor, determine the convection coefficient.

SCHEMATIC:



ASSUMPTIONS: (1) Electrical power is dissipated uniformly within the resistor, (2) Temperature of the resistor is uniform, (3) Negligible electrical power dissipated in the lead wires, (4) Negligible radiation exchange between the resistor and the surroundings, (5) No heat transfer occurs from the battery, (5) Steady-state conditions in the resistor.

ANALYSIS: (a) Referring to Section 1.3.1, the conservation of energy requirement for a control volume at an instant of time, Equation 1.12c, is

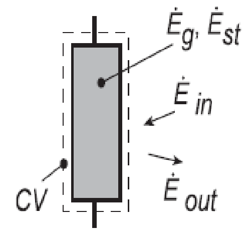
$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st}$$

where $\dot{E}_{in}$, $\dot{E}_{out}$ correspond to *surface* inflow and outflow processes, respectively. The energy generation term $\dot{E}_g$ is associated with conversion of some other energy form (chemical, electrical, electromagnetic or nuclear) to thermal energy. The energy storage term $\dot{E}_{st}$ is associated with changes in the internal, kinetic and/or potential energies of the matter in the control volume. $\dot{E}_g$, $\dot{E}_{st}$ are *volumetric* phenomena. The electrical power delivered by the battery is P = VI = 24V×6A = 144 W.

Control volume: Resistor.

$$\dot{E}_g = 144\,W \qquad \dot{E}_{st} = 0$$

$$\dot{E}_{in} = 0 \qquad \dot{E}_{out} = 144\,W \qquad <$$

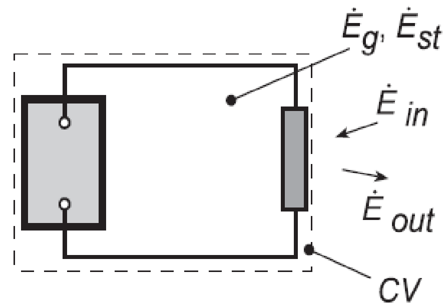


The $\dot{E}_g$ term is due to conversion of electrical energy to thermal energy. The term $\dot{E}_{out}$ is due to convection from the resistor surface to the air.

Continued...

PROBLEM 1.29 (Cont.)

Control volume: Battery-Resistor System.



$\dot{E}_{in} = 0 \qquad \dot{E}_{out} = 144\,\text{W}$ <

$\dot{E}_g = 144\text{W} \qquad \dot{E}_{st} = 0$

Since we are considering conservation of thermal and mechanical energy, the conversion of chemical energy to electrical energy in the battery is irrelevant, and including the battery in the control volume doesn't change the thermal and mechanical energy terms

(b) From the energy balance on the resistor with volume, $\forall = (\pi D^2/4)L$,

$$\dot{E}_g = \dot{q}\forall \qquad 144\,\text{W} = \dot{q}\left(\pi\left(0.06\,\text{m}\right)^2/4\right)\times 0.25\,\text{m} \qquad \dot{q} = 2.04\times10^5\,\text{W/m}^3$$ <

(c) From the energy balance on the resistor and Newton's law of cooling with $A_s = \pi DL + 2(\pi D^2/4)$,

$$\dot{E}_{out} = q_{cv} = hA_s\left(T_s - T_\infty\right)$$

$$144\,\text{W} = h\left[\pi\times0.06\,\text{m}\times0.25\,\text{m} + 2\left(\pi\times0.06^2\,\text{m}^2/4\right)\right]\left(95-25\right)^\circ\text{C}$$

$$144\,\text{W} = h\left[0.0471 + 0.0057\right]\text{m}^2\left(95-25\right)^\circ\text{C}$$

$$h = 39.0\,\text{W/m}^2\cdot\text{K}$$ <

COMMENTS: (1) In using the conservation of energy requirement, Equation 1.12c, it is important to recognize that $\dot{E}_{in}$ and $\dot{E}_{out}$ will always represent *surface* processes and $\dot{E}_g$ and $\dot{E}_{st}$, *volumetric* processes. The generation term $\dot{E}_g$ is associated with a *conversion* process from some form of energy to *thermal energy*. The storage term $\dot{E}_{st}$ represents the rate of change of *internal kinetic, and potential energy*.

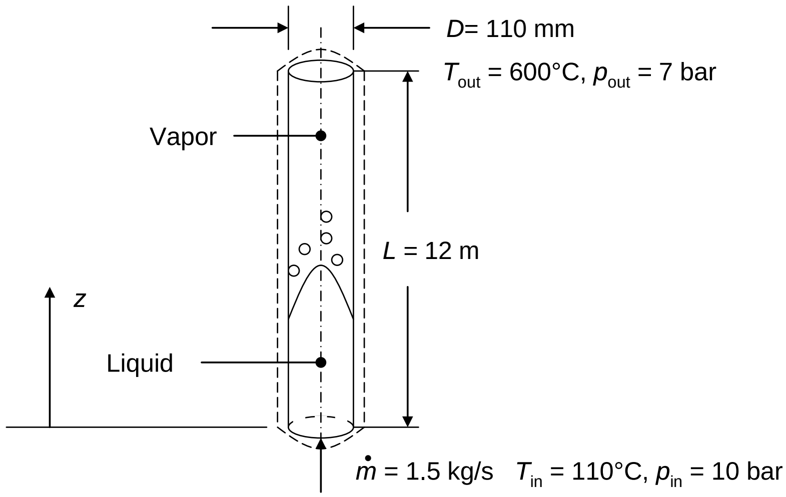
(2) From Table 1.1 and the magnitude of the convection coefficient determined from part (c), we conclude that the resistor is experiencing forced, rather than free, convection.

PROBLEM 1.30

KNOWN: Inlet and outlet conditions for flow of water in a vertical tube.

FIND: (a) Change in combined thermal and flow work, (b) change in mechanical energy, and (c) change in total energy of the water from the inlet to the outlet of the tube, (d) heat transfer rate, *q*.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform velocity distributions at the tube inlet and outlet.

PROPERTIES: Table A.6 water (T = 110°C): ρ = 950 kg/m^3, (T = (179.9°C + 110 °C)/2 = 145°C): c_p = 4300 J/kg·K, ρ = 919 kg/m^3. Other properties are taken from Moran, M.J. and Shapiro, H.N., *Fundamentals of Engineering Thermodynamics*, 6th Edition, John Wiley & Sons, Hoboken, 2008 including (p_{sat} = 10 bar): T_{sat} = 179.9°C, i_f = 762.81 kJ/kg; (p = 7 bar, T = 600°C): i = 3700.2 kJ/kg, υ = 0.5738 m^3/kg.

ANALYSIS: The steady-flow energy equation, in the absence of work (other than flow work), is

$$\begin{aligned} &\dot{m}\left(u + pv + \tfrac{1}{2}V^2 + gz\right)_{\text{in}} - \dot{m}\left(u + pv + \tfrac{1}{2}V^2 + gz\right)_{\text{out}} + q = 0 \\ &\dot{m}\left(i + \tfrac{1}{2}V^2 + gz\right)_{\text{in}} - \dot{m}\left(i + \tfrac{1}{2}V^2 + gz\right)_{\text{out}} + q = 0 \end{aligned} \qquad (1)$$

while the conservation of mass principle yields

$$V_{\text{in}} = \frac{4\dot{m}}{\rho\pi D^2} = \frac{4\times 1.5\text{ kg/s}}{950\text{ kg/m}^3\times\pi\times(0.110\text{ m})^2} = 0.166\text{ m/s}\ ;\ V_{\text{out}} = \frac{\upsilon 4\dot{m}}{\pi D^2} = \frac{0.5738\text{ m}^3/\text{kg}\times 4\times 1.5\text{ kg/s}}{\pi\times(0.110\text{ m})^2} = 90.6\text{ m/s}$$

(a) The change in the combined thermal and flow work energy from inlet to outlet:

$$\begin{aligned} E_{i,\text{out}} - E_{i,\text{in}} &= \dot{m}(i)_{\text{out}} - \dot{m}(i)_{\text{in}} = \dot{m}(i)_{\text{out}} - \dot{m}[i_{f,\text{sat}} + c_p(T_{\text{in}} - T_{\text{sat}})] \\ &= 1.5\text{ kg/s}\times\left(3700.2\text{ kJ/kg} - [762.81\text{ kJ/kg} + 4.3\text{ kJ/kg}\cdot\text{K}\times(110 - 179.9)°\text{C}]\right) \quad < \\ &= 4.86\text{ MW} \end{aligned}$$

where $i_{f,sat}$ is the enthalpy of saturated liquid at the phase change temperature and pressure.

(b) The change in mechanical energy from inlet to outlet is:

Continued…

PROBLEM 1.30 (cont.)

$$E_{m,\text{out}} - E_{m,\text{in}} = \dot{m}(\tfrac{1}{2}V^2 + gz)_{\text{out}} - \dot{m}(\tfrac{1}{2}V^2 + gz)_{\text{in}}$$

$$= 1.5\ \text{kg/s} \times \left(\tfrac{1}{2}\left[(90.6\ \text{m/s})^2 - (0.166\ \text{m/s})^2 \right] + 9.8\ \text{m/s}^2 \times 12\ \text{m} \right) = 6.33\ \text{kW} \qquad <$$

(c) The change in the total energy is the summation of the thermal, flow work, and mechanical energy change or

$$E_{\text{in}} - E_{\text{out}} = 4.86\ \text{MW} + 6.33\ \text{kW} = 4.87\ \text{MW} \qquad <$$

(d) The total heat transfer rate is the same as the total energy change, $q = E_{\text{in}} - E_{\text{out}} = 4.87\ \text{MW}$ $<$

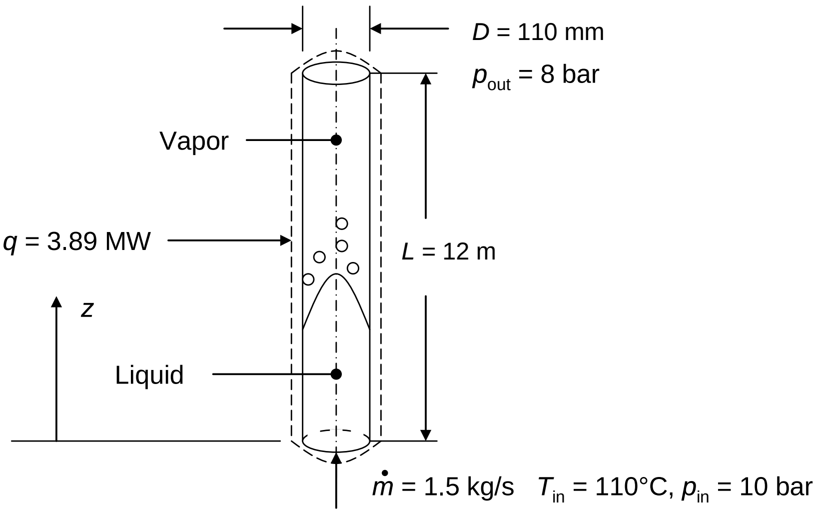
COMMENTS: (1) The change in mechanical energy, consisting of kinetic and potential energy components, is negligible compared to the change in thermal and flow work energy. (2) The average heat flux at the tube surface is $q'' = q/(\pi DL) = 4.87\text{MW}/(\pi \times 0.110\ \text{m} \times 12\ \text{m}) = 1.17\ \text{MW/m}^2$, which is very large. (3) The change in the velocity of the water is inversely proportional to the change in the density. As such, the outlet velocity is very large, and large pressure drops will occur in the vapor region of the tube relative to the liquid region of the tube.

PROBLEM 1.31

KNOWN: Flow of water in a vertical tube. Tube dimensions. Mass flow rate. Inlet pressure and temperature. Heat rate. Outlet pressure.

FIND: (a) Outlet temperature, (b) change in combined thermal and flow work, (c) change in mechanical energy, and (d) change in total energy of the water from the inlet to the outlet of the tube.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible change in mechanical energy. (3) Uniform velocity distributions at the tube inlet and outlet.

PROPERTIES: Table A.6 water (T = 110°C): ρ = 950 kg/m^3, (T = (179.9°C + 110 °C)/2 = 145°C): c_p = 4300 J/kg·K, ρ = 919 kg/m^3. Other properties are taken from Moran, M.J. and Shapiro, H.N., *Fundamentals of Engineering Thermodynamics*, 6th Edition, John Wiley & Sons, Hoboken, 2008 including (p_{sat} = 10 bar): T_{sat} = 179.9°C, i_f = 762.81 kJ/kg; (p = 8 bar, i = 3056 kJ/kg): T = 300°C, υ = 0.3335 m^3/kg.

ANALYSIS: (a) The steady-flow energy equation, in the absence of work (other than flow work), is

$$\dot{m}\left(u + pv + \tfrac{1}{2}V^2 + gz\right)_{\text{in}} - \dot{m}\left(u + pv + \tfrac{1}{2}V^2 + gz\right)_{\text{out}} + q = 0$$
$$\dot{m}\left(i + \tfrac{1}{2}V^2 + gz\right)_{\text{in}} - \dot{m}\left(i + \tfrac{1}{2}V^2 + gz\right)_{\text{out}} + q = 0 \qquad (1)$$

Neglecting the change in mechanical energy yields

$$\dot{m}(i_{\text{in}} - i_{\text{out}}) + q = 0$$

The inlet enthalpy is

$$i_{\text{in}} = i_{f,\text{in}} + c_p(T_{\text{in}} - T_{\text{sat}}) = 762.81\ \text{kJ/kg} + 4.3\ \text{kJ/kg}\cdot\text{K} \times (110 - 179.9)°\text{C} = 462.2\ \text{kJ/kg}$$

Thus the outlet enthalpy is

$$i_{\text{out}} = i_{\text{in}} + q/\dot{m} = 462.2\ \text{kJ/kg} + 3890\ \text{kW}/1.5\ \text{kg/s} = 3056\ \text{kJ/kg}$$

Continued…

and the outlet temperature can be found from thermodynamic tables at p = 8 bars, i = 3056 kJ/kg, for which

$$T_{out} = 300°C \qquad <$$

(b) The change in the combined thermal and flow work energy from inlet to outlet:

$$E_{i,\text{out}} - E_{i,\text{in}} = \dot{m}(i_{\text{out}} - i_{\text{in}}) = q = 3.89 \text{ MW} \qquad <$$

(c) The change in mechanical energy can now be calculated. First, the outlet specific volume can be found from thermodynamic tables at T_{out} = 300°C, p_{out} = 8 bars, υ = 0.3335 m^3/kg. Next, the conservation of mass principle yields

$$V_{\text{in}} = \frac{4\dot{m}}{\rho\pi D^2} = \frac{4 \times 1.5 \text{ kg/s}}{950 \text{ kg/m}^3 \times \pi \times (0.110 \text{ m})^2} = 0.166 \text{ m/s} \text{ ; } V_{\text{out}} = \frac{\upsilon 4\dot{m}}{\pi D^2} = \frac{0.3335 \text{ m}^3/\text{kg} \times 4 \times 1.5 \text{ kg/s}}{\pi \times (0.110 \text{ m})^2} = 52.6 \text{ m/s}$$

The change in mechanical energy from inlet to outlet is:

$$\begin{aligned} E_{m,\text{out}} - E_{m,\text{in}} &= \dot{m}(\tfrac{1}{2}V^2 + gz)_{\text{out}} - \dot{m}(\tfrac{1}{2}V^2 + gz)_{\text{in}} \\ &= 1.5 \text{ kg/s} \times \left(\tfrac{1}{2}\left[(52.6 \text{ m/s})^2 - (0.166 \text{ m/s})^2\right] + 9.8 \text{ m/s}^2 \times 12 \text{ m}\right) = 2.25 \text{ kW} \end{aligned} \qquad <$$

(d) The change in the total energy is the summation of the thermal, flow work, and mechanical energy change or

$$E_{in} - E_{out} = 3.89 \text{ MW} + 2.25 \text{ kW} = 3.89 \text{ MW} \qquad <$$

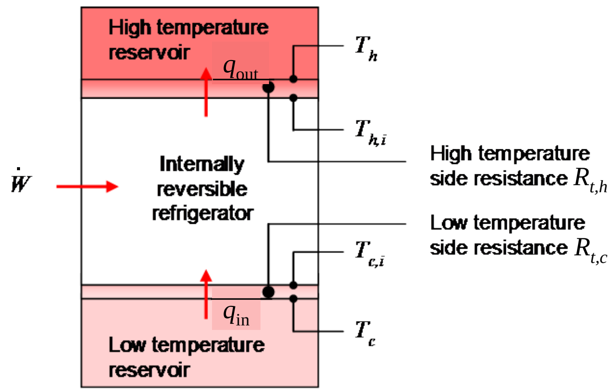
COMMENTS: (1) The change in mechanical energy, consisting of kinetic and potential energy components, is negligible compared to the change in thermal and flow work energy. (2) The average heat flux at the tube surface is $q'' = q/(\pi DL) = 3.89 \text{ MW}/(\pi \times 0.110 \text{ m} \times 12 \text{ m}) = 0.94 \text{ MW/m}^2$, which is very large. (3) The change in the velocity of the water is inversely proportional to the change in the density. As such, the outlet velocity is very large, and large pressure drops will occur in the vapor region of the tube relative to the liquid region of the tube.

PROBLEM 1.32

KNOWN: Hot and cold reservoir temperatures of an internally reversible refrigerator. Thermal resistances between refrigerator and hot and cold reservoirs.

FIND: Expressions for modified Coefficient of Performance and power input of refrigerator.

SCHEMATIC:



ASSUMPTIONS: (1) Refrigerator is internally reversible, (2) Steady-state operation.

ANALYSIS: Heat is transferred from the low temperature reservoir (the refrigerated space) at T_c to the refrigerator unit, through the resistance $R_{t,c}$, with $T_c > T_{c,i}$. Heat is rejected from the refrigerator unit to the higher temperature reservoir (the surroundings), through the resistance $R_{t,h}$, with $T_{h,i} > T_h$. The heat input and output rates can be expressed in a manner analogous to Equations 1.18a and 1.18b.

$$q_{\text{in}} = (T_c - T_{c,i}) / R_{t,c} \tag{1}$$

$$q_{\text{out}} = (T_{h,i} - T_h) / R_{t,h} \tag{2}$$

Equations (1) and (2) can be solved for the internal temperatures, to yield

$$T_{h,i} = T_h + q_{\text{out}} R_{t,h} = T_h + q_{\text{in}} R_{t,h} \left(\frac{1 + \text{COP}_m}{\text{COP}_m} \right) \tag{3}$$

$$T_{c,i} = T_c - q_{\text{in}} R_{t,c} \tag{4}$$

In Equation (3), q_{out} has been expressed as

$$q_{\text{out}} = q_{\text{in}} \left(\frac{1 + \text{COP}_m}{\text{COP}_m} \right) \tag{5}$$

using the definition of COP_m given in the problem statement. The modified Coefficient of Performance can then be expressed as

Continued…

$$\text{COP}_m = \frac{T_{c,i}}{T_{h,i} - T_{c,i}} = \frac{T_c - q_{\text{in}} R_{t,c}}{T_h + q_{\text{in}} R_{t,h}\left(\dfrac{1+\text{COP}_m}{\text{COP}_m}\right) - T_c + q_{\text{in}} R_{t,c}}$$

Manipulating this expression,

$$\left(T_h - T_c + q_{\text{in}} R_{t,c}\right)\text{COP}_m + q_{\text{in}} R_{t,h}(1+\text{COP}_m) = T_c - q_{\text{in}} R_{t,c}$$

Solving for COP_m results in

$$\text{COP}_m = \frac{T_c - q_{\text{in}} R_{\text{tot}}}{T_h - T_c + q_{\text{in}} R_{\text{tot}}}$$ <

From the definition of COP_m, the power input can be determined:

$$\dot{W} = \frac{q_{\text{in}}}{\text{COP}_m} = q_{\text{in}} \frac{T_h - T_c + q_{\text{in}} R_{\text{tot}}}{T_c - q_{\text{in}} R_{\text{tot}}}$$ <

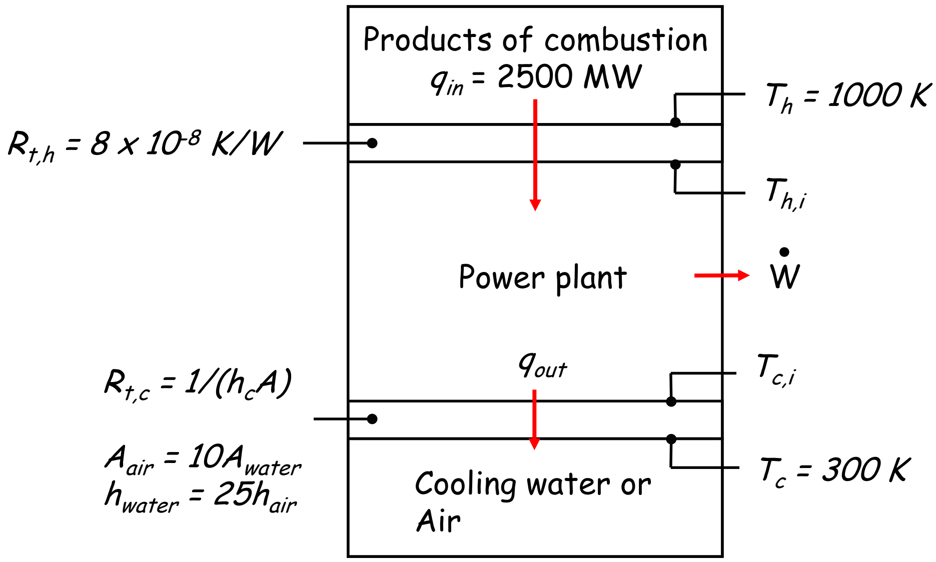
COMMENTS: As q_{in} or R_{tot} goes to zero, the Coefficient of Performance approaches the maximum Carnot value, $\text{COP}_m = \text{COP}_C = T_c /(T_h - T_c)$.

PROBLEM 1.33

KNOWN: Power plant and operating conditions of Example 1.7. Change in cold-side heat transfer surface area and convection heat transfer coefficient.

FIND: Modified efficiency and power output.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) power plant operates as an internally reversible heat engine, (3) clean operating conditions.

ANALYSIS: The cold-side thermal resistance for water cooling (for design conditions) is provided in Example 1.7 and is $R_{t,c} = 2 \times 10^{-8}$ K/W. The cold side thermal resistance is given by $R_{t,c} = 1/(h_c A)$, therefore

$$\frac{R_{t,c,\text{air}}}{R_{t,c,\text{water}}} = \frac{(hA)_{\text{water}}}{(hA)_{\text{air}}} = \left(\frac{h_{\text{water}}}{h_{\text{air}}}\right) \times \left(\frac{A_{\text{water}}}{A_{\text{air}}}\right) = \frac{25}{10} = 2.5$$

Hence, $R_{t,c,\text{air}} = 2.5 \times 2 \times 10^{-8}$ K/W $= 5 \times 10^{-8}$ K/W and $R_{\text{tot,air}} = 8 \times 10^{-8} + 5 \times 10^{-8} = 13 \times 10^{-8}$ K/W.

The modified efficiency for the air-cooled condenser is

$$\eta_m = 1 - \frac{T_c}{T_h - q_{in} R_{\text{tot,air}}} = 1 - \frac{300\text{K}}{1000\text{K} - 2500 \times 10^6\,\text{W} \times 1.3 \times 10^{-7}\,\text{K/W}} = 0.556 \qquad <$$

The power output is

$$\dot{W} = q_{\text{in}} \eta_m = 2500\text{ MW} \times 0.556 = 1390\text{ MW} \qquad <$$

Continued …

PROBLEM 1.33 (Cont.)

The air-cooled condenser is both (1) more expensive and (2) leads to a lower plant efficiency and power output relative to the water-cooled condenser of Example 1.7.

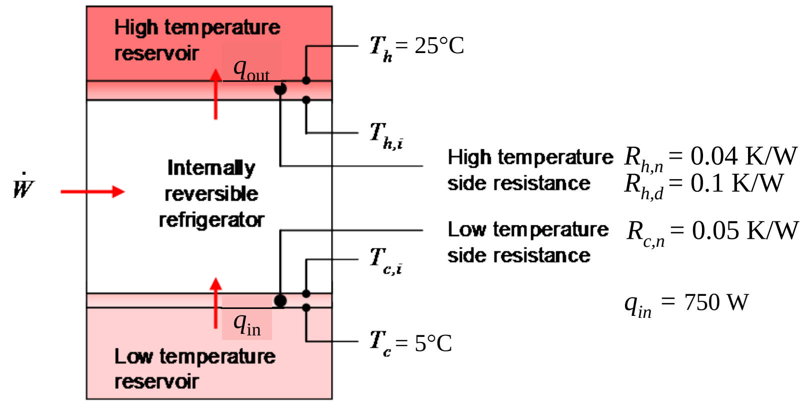
COMMENT: The diminished performance and higher cost of the air-cooled condenser, relative to the water-cooled condenser, is typical. This problem illustrates the profound linkage between power generation and water usage, and is referred to as "the water-energy nexus."

PROBLEM 1.34

KNOWN: Hot and cold reservoir temperatures of an internally reversible refrigerator. Thermal resistances between refrigerator and hot and cold reservoirs under clean and dusty conditions. Desired cooling rate.

FIND: Modified Coefficient of Performance and power input of refrigerator under clean and dusty conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Refrigerator is internally reversible, (2) Steady-state operation, (3) Cold side thermal resistance does not degrade over time.

ANALYSIS: According to Problem 1.32, the modified Coefficient of Performance and power input are given by

$$\mathrm{COP}_m = \frac{T_c - q_{\mathrm{in}} R_{\mathrm{tot}}}{T_h - T_c + q_{\mathrm{in}} R_{\mathrm{tot}}} \tag{1}$$

$$\dot{W} = q_{\mathrm{in}} \frac{T_h - T_c + q_{\mathrm{in}} R_{\mathrm{tot}}}{T_c - q_{\mathrm{in}} R_{\mathrm{tot}}} \tag{2}$$

Under new, clean conditions, with $R_{\mathrm{tot},n} = R_{h,n} + R_{c,n} = 0.09$ K/W, we find

$$\mathrm{COP}_{m,n} = \frac{278\ \mathrm{K} - 750\ \mathrm{W} \times 0.09\ \mathrm{K/W}}{298\ \mathrm{K} - 278\ \mathrm{K} + 750\ \mathrm{W} \times 0.09\ \mathrm{K/W}} = 2.41$$ <

$$\dot{W}_n = 750\ \mathrm{W} \frac{298\ \mathrm{K} - 278\ \mathrm{K} + 750\ \mathrm{W} \times 0.09\ \mathrm{K/W}}{278\ \mathrm{K} - 750\ \mathrm{W} \times 0.09\ \mathrm{K/W}} = 312\ \mathrm{W}$$ <

Under dusty, conditions, with $R_{\mathrm{tot},d} = R_{h,d} + R_{c,n} = 0.15$ K/W, we find

$$\mathrm{COP}_{m,d} = \frac{278\ \mathrm{K} - 750\ \mathrm{W} \times 0.15\ \mathrm{K/W}}{298\ \mathrm{K} - 278\ \mathrm{K} + 750\ \mathrm{W} \times 0.15\ \mathrm{K/W}} = 1.25$$ <

Continued...

$$\dot{W}_d = 750\text{ W}\,\frac{298\text{ K} - 278\text{ K} + 750\text{ W} \times 0.15\text{ K/W}}{278\text{ K} - 750\text{ W} \times 0.15\text{ K/W}} = 600\text{ W} \qquad <$$

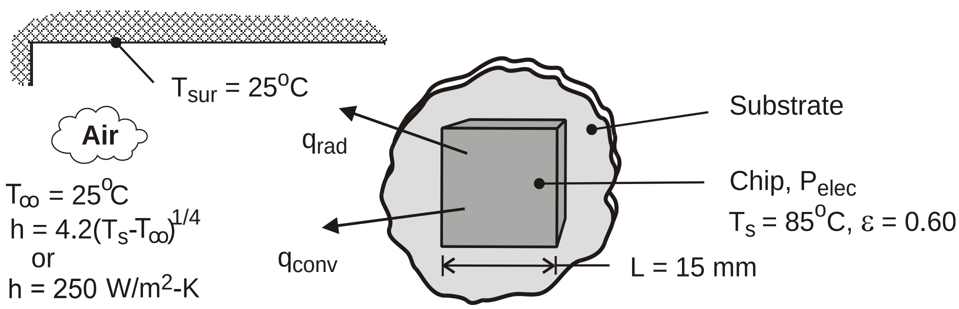
COMMENTS: (1) The cooling rates and power input values are time-averaged quantities. Since the refrigerator does not run constantly, the instantaneous power requirements would be higher than calculated. (2) In practice, when the condenser coils become dusty the power input does not adjust to maintain the cooling rate. Rather, the refrigerator's *duty cycle* would increase. (3) The ideal Carnot Coefficient of Performance is $COP_C = T_c/(T_h - T_c) = 14$ and the corresponding power input is 54 W. (4) This refrigerator's energy efficiency is poor. Less power would be consumed by more thoroughly insulating the refrigerator, and designing the refrigerator to minimize heat gain upon opening its door, in order to reduce the cooling rate, q_{in}.

PROBLEM 1.35

KNOWN: Width, surface emissivity and maximum allowable temperature of an electronic chip. Temperature of air and surroundings. Convection coefficient.

FIND: (a) Maximum power dissipation for free convection with $h(W/m^2 \cdot K) = 4.2(T - T_\infty)^{1/4}$, (b) Maximum power dissipation for forced convection with $h = 250\ W/m^2 \cdot K$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Radiation exchange between a small surface and a large enclosure, (3) Negligible heat transfer from sides of chip or from back of chip by conduction through the substrate.

ANALYSIS: Subject to the foregoing assumptions, electric power dissipation by the chip must be balanced by convection and radiation heat transfer from the chip. Hence, from Eq. (1.10),

$$P_{elec} = q_{conv} + q_{rad} = hA\left(T_s - T_\infty\right) + \varepsilon A \sigma \left(T_s^4 - T_{sur}^4\right)$$

where $A = L^2 = \left(0.015m\right)^2 = 2.25\times10^{-4} m^2.$

(a) If heat transfer is by natural convection,

$$q_{conv} = C\ A\left(T_s - T_\infty\right)^{5/4} = 4.2\ W/m^2 \cdot K^{5/4}\left(2.25\times10^{-4} m^2\right)\left(60K\right)^{5/4} = 0.158\ W$$

$$q_{rad} = 0.60\left(2.25\times10^{-4} m^2\right)5.67\times10^{-8}\ W/m^2 \cdot K^4\left(358^4 - 298^4\right)K^4 = 0.065\ W$$

$$P_{elec} = 0.158\ W + 0.065\ W = 0.223\ W$$ <

(b) If heat transfer is by forced convection,

$$q_{conv} = hA\left(T_s - T_\infty\right) = 250\ W/m^2 \cdot K\left(2.25\times10^{-4} m^2\right)\left(60K\right) = 3.375\ W$$

$$P_{elec} = 3.375\ W + 0.065\ W = 3.44\ W$$ <

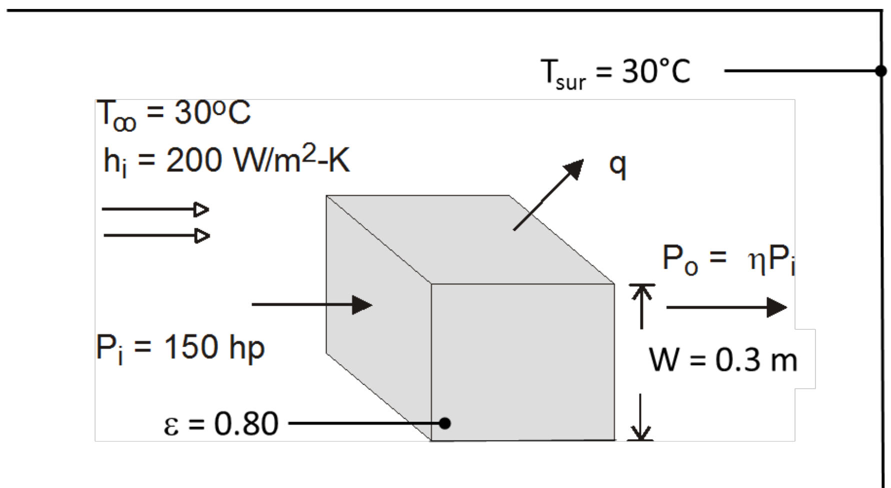
COMMENTS: Clearly, radiation and natural convection are inefficient mechanisms for transferring heat from the chip. For T_s = 85°C and T_∞ = 25°C, the natural convection coefficient is 11.7 $W/m^2 \cdot K$. Even for forced convection with $h = 250\ W/m^2 \cdot K$, the power dissipation is well below that associated with many of today's processors. To provide acceptable cooling, it is often necessary to attach the chip to a highly conducting substrate and to thereby provide an additional heat transfer mechanism due to conduction from the back surface.

PROBLEM 1.36

KNOWN: Width, input power, and efficiency of a transmission. Temperature and convection coefficient for air flow over the casing. Emissivity of casing and temperature of surroundings.

FIND: Surface temperature of casing. Resistances due to convection and radiation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady state, (2) Uniform convection coefficient and surface temperature, (3) Radiation exchange with large surroundings.

ANALYSIS: Heat transfer from the case must balance heat dissipation in the transmission, which may be expressed as $q = P_i - P_o = P_i\,(1 - \eta)$ = 150 hp × 746 W/hp × 0.07 = 7833 W. Heat transfer from the case is by convection and radiation, in which case

$$q = A_s\left[h\left(T_s - T_\infty\right) + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]$$

where $A_s = 6\ W^2$. Hence,

$$7833\,\mathrm{W} = 6\left(0.30\,\mathrm{m}\right)^2\left[200\,\mathrm{W/m^2\cdot K}\left(T_s - 303\mathrm{K}\right) + 0.8 \times 5.67 \times 10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(T_s^4 - 303^4\right)\mathrm{K}^4\right]$$

A trial-and-error solution yields

$$T_s \approx 373\,\mathrm{K} = 100°\mathrm{C}$$ <

The thermal resistances can be found from Eq. 1.11, that is, $R_t = \Delta T/q$. For convection, the relevant temperature difference is $T_s - T_\infty$.

$$R_{t,conv} = \left(T_s - T_\infty\right)/q_{conv} = \left(T_s - T_\infty\right)/\left[hA_s\left(T_s - T_\infty\right)\right] = 1/hA_s$$

$$R_{t,conv} = 1/\left[6\left(0.30\,\mathrm{m}\right)^2 200\,\mathrm{W/m^2\cdot K}\right] = 0.00926\ \mathrm{K/W}$$ <

For radiation, the relevant temperature difference is $T_s - T_{sur}$ and

Continued …

PROBLEM 1.36 (Cont.)

$$R_{t,rad} = \left(T_s - T_{sur}\right)/\, q_{rad} = \left(T_s - T_{sur}\right)/\left[\varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)A_s\right]$$

$$R_{t,rad} = \left(373 - 303\right)K/\left[0.8\times5.67\times10^{-8}\ W/m^2\cdot K^4\left(373^4 - 303^4\right)K^4\, 6\left(0.30\,m\right)^2\right] \qquad <$$

$$= 0.262\ K/W$$

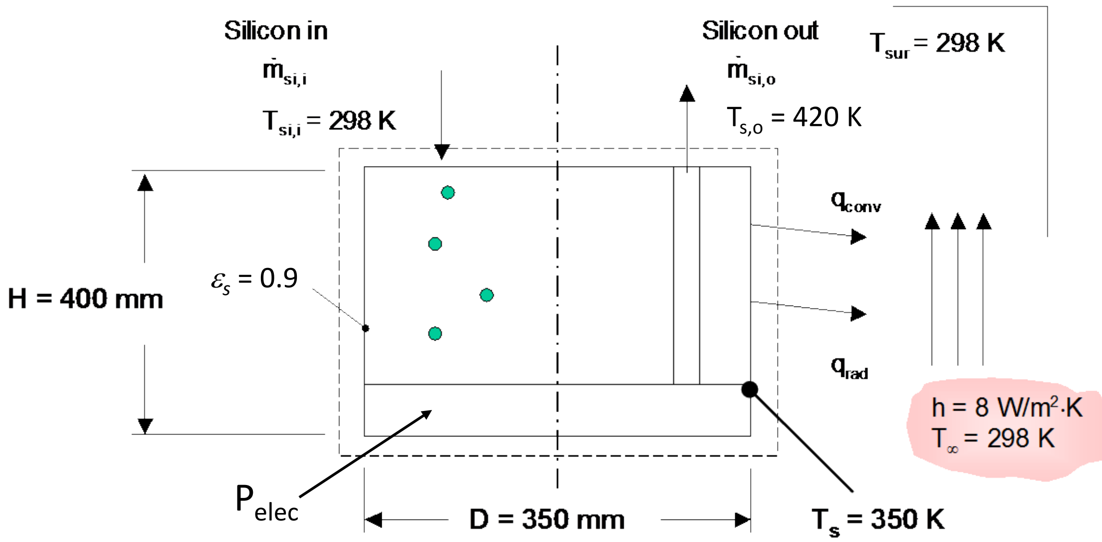
COMMENTS: (1) For $T_s \approx 373$ K, $q_{conv} \approx 7{,}560$ W and $q_{rad} \approx 270$ W, in which case heat transfer is dominated by convection. This can also be seen by the fact that the resistance to radiation heat transfer is much larger than the resistance to convection heat transfer. (2) If radiation is neglected, the corresponding surface temperature is $T_s = 102.5°C$.

PROBLEM 1.37

KNOWN: Process for growing thin, photovoltaic grade silicon sheets. Sheet dimensions and velocity. Dimensions, surface temperature and surface emissivity of growth chamber. Surroundings and ambient temperatures, and convective heat transfer coefficient. Amount of time-averaged absorbed solar irradiation and photovoltaic conversion efficiency.

FIND: (a) Electric power needed to operate at steady state, (b) Time needed to operate the photovoltaic panel to produce enough energy to offset energy consumed during its manufacture.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Large surroundings, (3) Constant properties, (4) Neglect the presence of the strings.

PROPERTIES: *Table A-1*, Silicon (T = 300 K): c = 712 J/kg·K, ρ = 2330 kg/m^3, (T = 420 K): c = 798 J/kg·K.

ANALYSIS: (a) At steady state, the mass of silicon produced per unit time is equal to the mass of silicon added to the system per unit time. The amount of silicon produced is

$$\dot{m} = W_{\text{si}} \times t_{\text{si}} \times V_{\text{si}} \times \rho = 0.075\text{m} \times 140 \times 10^{-6}\text{m} \times 0.018\text{m/min} \times (1/60)\text{min/s} \times 2330\text{kg/m}^3 = 7.34 \times 10^{-6}\text{kg/s}$$

At steady state, $\dot{E}_{in} = \dot{E}_{out}$ where

$$\dot{E}_{\text{in}} = P_{\text{elec}} + \dot{m}cT_{\text{si},i} = P_{\text{elec}} + 7.34 \times 10^{-6}\frac{\text{kg}}{\text{s}} \times 712\frac{\text{J}}{\text{kg}\cdot\text{K}} \times 298\text{K} = P_{\text{elec}} + 2.10\text{ J/s} = P_{\text{elec}} + 1.56\text{ W}$$

and

$$\dot{E}_{\text{out}} = \dot{m}cT_{\text{si},o} + \left[2\pi D^2/4 + H\pi D\right]\left[h(T_s - T_\infty) + \varepsilon\sigma(T_s^4 - T_{\text{sur}}^4)\right]$$

$$= 7.34 \times 10^{-6}\frac{\text{kg}}{\text{s}} \times 798\frac{\text{J}}{\text{kg}\cdot\text{K}} \times 420\text{K} + \left[2\pi \times (0.35\text{m})^2/4 + 0.4\text{m} \times \pi \times 0.35\text{m}\right]$$

$$\times\left[8\frac{\text{W}}{\text{m}^2\cdot\text{K}} \times (350-298)\text{K} + 5.67 \times 10^{-8}\frac{\text{W}}{\text{m}^2\cdot\text{K}^4} \times 0.9 \times \left((350\text{K})^4 - (298\text{K})^4\right)\right] = 493\text{ W}$$

Therefore,

Continued…

PROBLEM 1.37 (Cont.)

$$P_{\text{elec}} = 493\text{ W} - 1.56\text{ W} = 491\text{ W} \qquad <$$

(b) The electric energy needed to manufacture the photovoltaic material is

$$E_m = P_{\text{elec}}/(W_s V_s) = 491\text{ W}/(0.075\text{ m} \times 0.018\text{ m/min} \times (1/60)\text{ min/s}) = 21.9 \times 10^6\text{ J/m}^2 = 21.9 \times 10^3\text{ kJ/m}^2$$

The time needed to generate E_m by the photovoltaic panel is

$$t = E_m / q''_{\text{sol}}\eta = 21.9 \times 10^6\text{ J/m}^2/(180\text{W}/\text{m}^2 \times 0.20) = 609 \times 10^3\text{s} = 169\text{ h} \qquad <$$

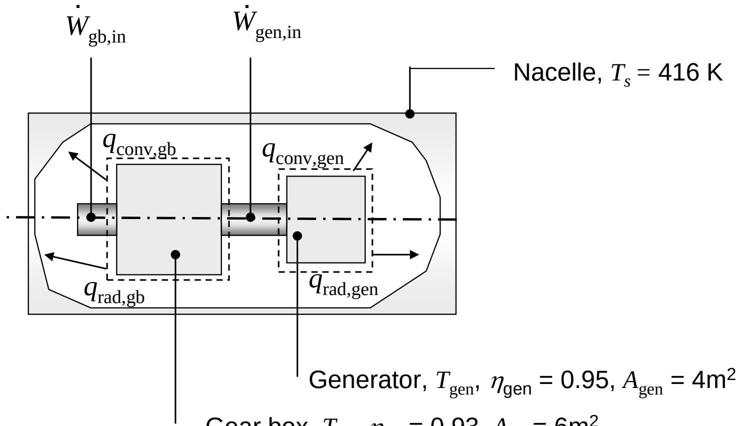
COMMENTS: (1) The radiation and convection losses are primarily responsible for the electric power needed to manufacture the photovoltaic material. Of these, radiation is responsible for 47% of the losses. The radiation losses could be reduced by coating the exposed surface of the chamber with a low emissivity material. (2) Assuming an electricity price of \$0.15/kWh, the cost of electricity to manufacture the material is 21.9×10^3 kJ/m^2 $\times$ (\$0.15/kWh) $\times$ (1h/3600s) = \$0.92/m^2. (3) This problem represents a type of *life cycle analysis* in which the energy consumed to manufacture a product is of interest. The analysis presented here does not account for the energy that is consumed to produce the silicon powder, the energy used to fabricate the growth chamber, or the energy that is used to fabricate and install the photovoltaic panel. The actual time needed to offset the energy to manufacture and install the photovoltaic panel will be much greater than 169 h. See Keoleian and Leis, “Application of Life-cycle Energy Analysis to Photovoltaic Module Design, *Progress in Photovoltaics: Research and Applications*, Vol. 5, pp. 287-300, 1997.

PROBLEM 1.38

KNOWN: Surface areas, convection heat transfer coefficient, surface emissivity of gear box and generator. Temperature of nacelle. Electric power generated by the wind turbine and generator efficiency.

FIND: Gear box and generator surface temperatures.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Interior of nacelle can be treated as large surroundings, (3) Negligible heat transfer between the gear box and the generator.

ANALYSIS: Heat is generated within both the gear box and the generator. The mechanical work into the generator can be determined from the electrical power, $P = 2.5 \times 10^6$ W, and the efficiency of the generator as

$$\dot{W}_{gen,in} = P / \eta_{gen} = 2.5 \times 10^6 \text{W} / 0.95 = 2.63 \times 10^6 \text{W}$$

Therefore, the heat transfer from the generator is

$$q_{gen} = \dot{W}_{gen} - P = 2.63 \times 10^6 \text{W} - 2.5 \times 10^6 \text{W} = 0.13 \times 10^6 \text{W}$$

The heat transfer is composed of convection and radiation components. Hence,

$$\begin{aligned} q_{gen} &= A_{gen}\left[h\left(T_{gen} - T_s\right) + \varepsilon\sigma\left(T_{gen}^4 - T_s^4\right)\right] \\ &= 4\text{m}^2 \times \left[40\frac{\text{W}}{\text{m}^2 \cdot \text{K}}\left(T_{gen} - 416\text{K}\right) + 0.90 \times 5.67 \times 10^{-8}\frac{\text{W}}{\text{m}^2 \cdot \text{K}^4}\left(T_{gen}^4 - \left(416\text{K}\right)^4\right)\right] \\ &= 0.13 \times 10^6 \text{W} \end{aligned}$$

The generator surface temperature may be found by using a numerical solver, or by trial-and-error, yielding

T_{gen} = 785 K = 512°C <

Continued...

PROBLEM 1.38 (Cont.)

Heat is also generated by the gear box. The heat generated in the gear box may be determined from knowledge of the heat generated cumulatively by the gear box and the generator, which is provided in Example 3.1 and is $q = q_{\text{gen}} + q_{\text{gb}} = 0.33 \times 10^6$ W. Hence, $q_{\text{gb}} = q - q_{\text{gen}} = 0.33 \times 10^6\text{W} - 0.13 \times 10^6\text{W} = 0.20 \times 10^6\text{W}$ and

$$
\begin{aligned}
q_{\text{gb}} &= A_{\text{gb}}\left[h\left(T_{\text{gb}} - T_s\right) + \varepsilon\sigma\left(T_{\text{gb}}^4 - T_s^4\right)\right] \\
&= 6\text{m}^2 \times \left[40\frac{\text{W}}{\text{m}^2 \cdot \text{K}}\left(T_{\text{gb}} - 416\text{K}\right) + 0.90 \times 5.67 \times 10^{-8}\frac{\text{W}}{\text{m}^2 \cdot \text{K}^4}\left(T_{\text{gb}}^4 - \left(416\text{K}\right)^4\right)\right] \\
&= 0.20 \times 10^6\text{W}
\end{aligned}
$$

which may be solved by trial-and-error or with a numerical solver to find

$$T_{\text{gb}} = 791 \text{ K} = 518°\text{C}$$ <

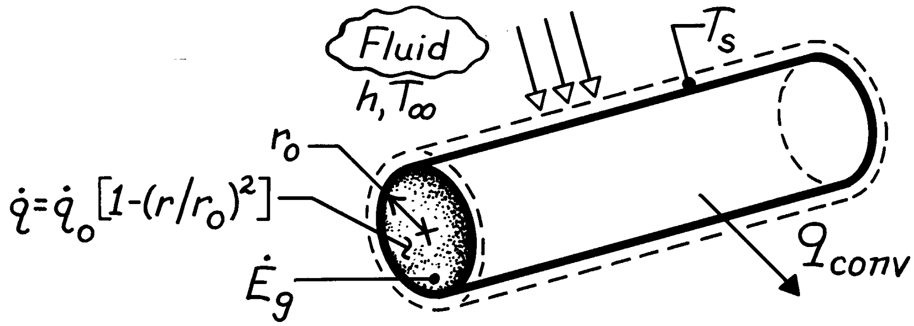
COMMENTS: (1) The gear box and generator temperatures are unacceptably high. Thermal management must be employed in order to generate power from the wind turbine. (2) The gear box and generator temperatures are of similar value. Hence, the assumption that heat transfer between the two mechanical devices is small is valid. (3) The radiation and convection heat transfer rates are of similar value. For the generator, convection and radiation heat transfer rates are $q_{\text{conv,gen}} = 5.9 \times 10^4$ W and $q_{\text{rad,gen}} = 7.1 \times 10^4$ W, respectively. The convection and radiation heat transfer rates are $q_{\text{conv,gb}} = 9.0 \times 10^4$ W and $q_{\text{rad,gb}} = 11.0 \times 10^4$ W, respectively, for the gear box. It would be a poor assumption to neglect either convection or radiation in the analysis.

PROBLEM 1.39

KNOWN: Radial distribution of heat dissipation in a cylindrical container of radioactive wastes. Surface convection conditions.

FIND: Total energy generation rate and surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible temperature drop across thin container wall.

ANALYSIS: The rate of energy generation is

$$\dot{E}_g = \int \dot{q}dV = \dot{q}_o \int_0^{r_o} \left[1-\left(r/r_o\right)^2\right] 2\pi rLdr$$

$$\dot{E}_g = 2\pi L\dot{q}_o\left(r_o^2/2 - r_o^2/4\right)$$

or per unit length,

$$\dot{E}'_g = \frac{\pi\dot{q}_o r_o^2}{2}. \qquad <$$

Performing an energy balance for a control surface about the container yields, at an instant,

$$\dot{E}'_g - \dot{E}'_{out} = 0$$

and substituting for the convection heat rate per unit length,

$$\frac{\pi\dot{q}_o r_o^2}{2} = h\left(2\pi r_o\right)\left(T_s - T_\infty\right)$$

$$T_s = T_\infty + \frac{\dot{q}_o r_o}{4h}. \qquad <$$

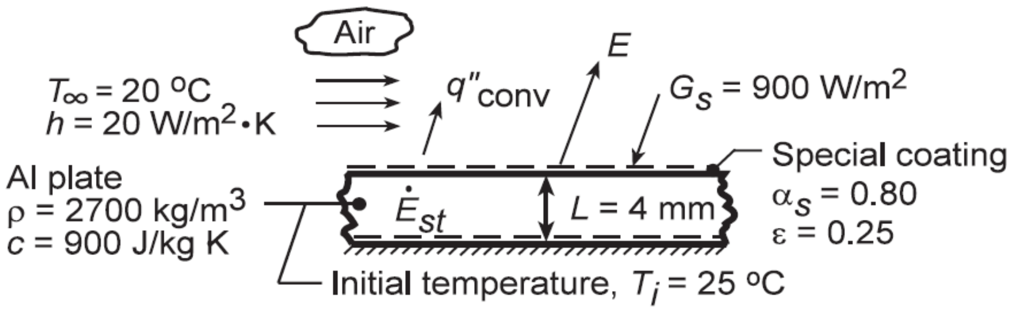
COMMENTS: The temperature within the radioactive wastes increases with decreasing r from T_s at r_o to a maximum value at the centerline.

PROBLEM 1.40

KNOWN: Thickness and initial temperature of an aluminum plate whose thermal environment is changed.

FIND: (a) Initial rate of temperature change, (b) Steady-state temperature of plate.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible end effects, (2) Uniform plate temperature at any instant, (3) Constant properties, (4) Adiabatic bottom surface, (5) Negligible radiation from surroundings, (6) No internal heat generation.

ANALYSIS: (a) Applying an energy balance, Eq. 1.12c, at an instant of time to a control volume about the plate, $\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$, it follows for a unit surface area.

$$\alpha_S G_S \left(1\,m^2\right) - E\left(1\,m^2\right) - q''_{conv}\left(1\,m^2\right) = (d/dt)(McT) = \rho\left(1\,m^2 \times L\right)c\,(dT/dt).$$

Rearranging and substituting from Eqs. 1.3 and 1.7, we obtain

$$dT/dt = (1/\rho Lc)\left[\alpha_S G_S - \varepsilon\sigma T_i^4 - h\left(T_i - T_\infty\right)\right].$$

$$dT/dt = \left(2700\,kg/m^3 \times 0.004\,m \times 900\,J/kg\cdot K\right)^{-1} \times$$

$$\left[0.8 \times 900\,W/m^2 - 0.25 \times 5.67 \times 10^{-8}\,W/m^2\cdot K^4\,(298\,K)^4 - 20\,W/m^2\cdot K\,(25-20)^\circ C\right]$$

$$dT/dt = 0.052^\circ\,C/s\,.$$ <

(b) Under steady-state conditions, $\dot{E}_{st}$ = 0, and the energy balance reduces to

$$\alpha_S G_S = \varepsilon\sigma T^4 + h\left(T - T_\infty\right) \qquad (2)$$

$$0.8 \times 900\,W/m^2 = 0.25 \times 5.67 \times 10^{-8}\,W/m^2\cdot K^4 \times T^4 + 20\,W/m^2\cdot K\,(T - 293\,K)$$

The solution yields T = 321.4 K = 48.4°C. <

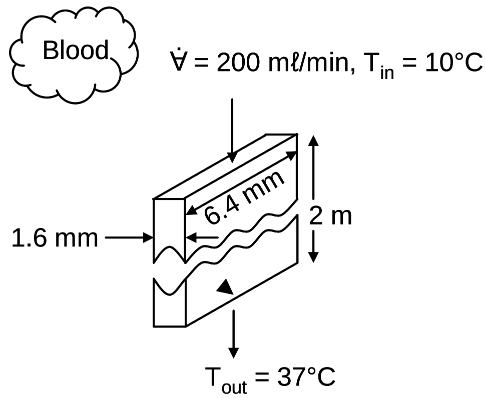
COMMENTS: The surface radiative properties have a significant effect on the plate temperature, which would decrease with increasing ε and decreasing α_S. If a low temperature is desired, the plate coating should be characterized by a large value of ε/α_S. The temperature would also decrease with increasing h.

PROBLEM 1.41

KNOWN: Blood inlet and outlet temperatures and flow rate. Dimensions of tubing.

FIND: Required rate of heat addition and estimate of kinetic and potential energy changes.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Incompressible liquid with negligible kinetic and potential energy changes, (3) Blood has properties of water.

PROPERTIES: Table A.6, Water ($\overline{T} \approx 300$ K): $c_{p,f} = 4179$ J/ kg· K, $\rho_f = 1/v_f = 997$ kg/m^3.

ANALYSIS: From an overall energy balance, Equation 1.12e,

$$q = \dot{m} c_p (T_{out} - T_{in})$$

where

$$\dot{m} = \rho_f \dot{\forall} = 997 \text{ kg/m}^3 \times 200 \text{ m}\ell\text{/min} \times 10^{-6} \text{ m}^3\text{/m}\ell \big/ 60 \text{ s/min} = 3.32 \times 10^{-3} \text{ kg/s}$$

Thus

$$q = 3.32 \times 10^{-3} \text{ kg/s} \times 4179 \text{ J/kg} \cdot \text{K} \times (37^\circ\text{C} - 10^\circ\text{C}) = 375 \text{ W}$$ <

The velocity in the tube is given by

$$V = \dot{\forall}/A_c = 200 \text{ m}\ell\text{/min} \times 10^{-6} \text{ m}^3\text{/m}\ell \big/ (60 \text{ s/min} \times 6.4 \times 10^{-3} \text{ m} \times 1.6 \times 10^{-3} \text{ m}) = 0.33 \text{ m/s}$$

The change in kinetic energy is

$$\dot{m}(\frac{1}{2}V^2 - 0) = 3.32 \times 10^{-3} \text{ kg/s} \times \frac{1}{2} \times (0.33 \text{ m/s})^2 = 1.8 \times 10^{-4} \text{ W}$$ <

The change in potential energy is

$$\dot{m} g z = 3.32 \times 10^{-3} \text{ kg/s} \times 9.8 \text{ m/s}^2 \times 2 \text{ m} = 0.065 \text{ W}$$ <

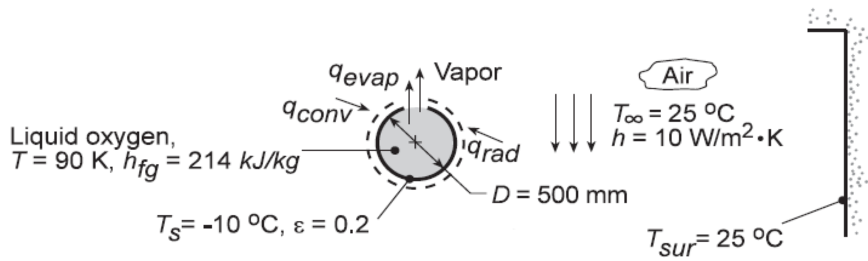
COMMENT: The kinetic and potential energy changes are both negligible relative to the thermal energy change.

PROBLEM 1.42

KNOWN: Boiling point and latent heat of liquid oxygen. Diameter and emissivity of container. Free convection coefficient and temperature of surrounding air and walls.

FIND: Mass evaporation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Temperature of container outer surface equals boiling point of oxygen.

ANALYSIS: (a) Applying mass and energy balances to a control surface about the container, it follows that, at any instant,

$$\frac{dm_{st}}{dt} = -\dot{m}_{out} = -\dot{m}_{evap} \qquad \frac{dE_{st}}{dt} = \dot{E}_{in} - \dot{E}_{out} = q_{conv} + q_{rad} - q_{evap}. \tag{1a,b}$$

With h_f as the enthalpy of liquid oxygen and h_g as the enthalpy of oxygen vapor, we have

$$E_{st} = m_{st} h_f \qquad q_{evap} = \dot{m}_{out} h_g \tag{2a,b}$$

Combining Equations (1a) and (2a,b), Equation (1b) becomes (with $h_{fg} = h_g - h_f$)

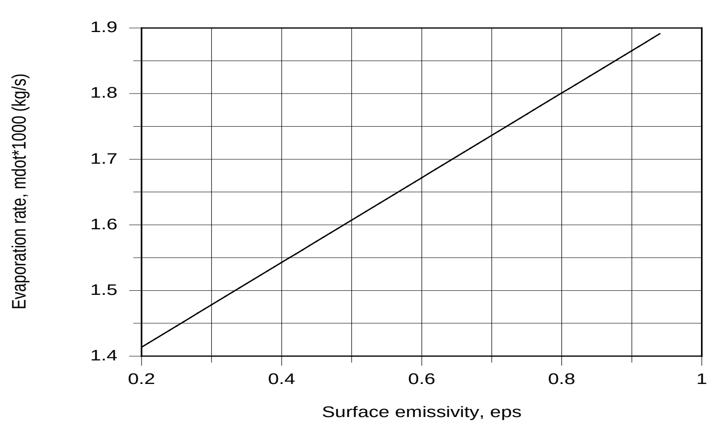
$$\dot{m}_{out} h_{fg} = q_{conv} + q_{rad}$$

$$\dot{m}_{evap} = \left(q_{conv} + q_{rad}\right)/h_{fg} = \left[h\left(T_\infty - T_s\right) + \varepsilon\sigma\left(T_{sur}^4 - T_s^4\right)\right]\pi D^2 / h_{fg} \tag{3}$$

$$\dot{m}_{evap} = \frac{\left[10\,\mathrm{W/m^2\cdot K}\left(298-263\right)\mathrm{K} + 0.2\times 5.67\times 10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(298^4 - 263^4\right)\mathrm{K}^4\right]\pi\left(0.5\,\mathrm{m}\right)^2}{214\,\mathrm{kJ/kg}}$$

$$\dot{m}_{evap} = \left(350 + 35.2\right)\mathrm{W/m^2}\left(0.785\ \mathrm{m^2}\right) / 214\,\mathrm{kJ/kg} = 1.41\times 10^{-3}\,\mathrm{kg/s}. \qquad <$$

(b) Using Equation (3), the mass rate of vapor production can be determined for the range of emissivity 0.2 to 0.94. The effect of increasing emissivity is to increase the heat rate into the container and, hence, increase the vapor production rate.



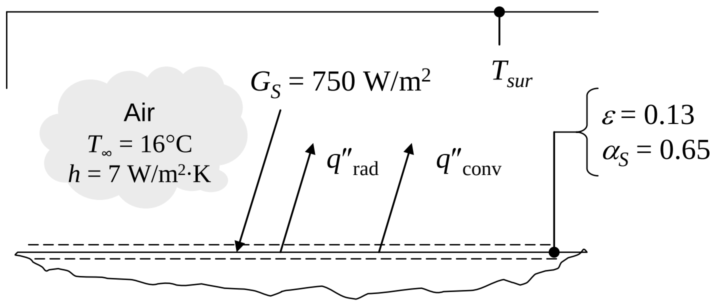
COMMENTS: To reduce the loss of oxygen due to vapor production, insulation should be applied to the outer surface of the container, in order to reduce q_{conv} and q_{rad}. Note from the calculations in part (a), that heat transfer by convection is greater than by radiation exchange.

PROBLEM 1.43

KNOWN: Emissivity and solar absorptivity of steel sheet. Solar irradiation, air temperature and convection coefficient.

FIND: Temperature of the steel sheet to determine cat comfort.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Bottom surface of steel is insulated, (3) Radiation from the environment can be treated as radiation from large surroundings, with $\alpha = \varepsilon$, (4) $T_{sur} = T_\infty$.

ANALYSIS: Performing a control surface energy balance on the top surface of the steel sheet gives (on a per unit area basis)

$$\alpha_S G_S - q''_{rad} - q''_{conv} = 0$$
$$\alpha_S G_S - \varepsilon\sigma(T_s^4 - T_{sur}^4) - h(T_s - T_\infty) = 0$$

$$0.65 \times 750\ \text{W/m}^2 - 0.13 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4 (T_s^4 - (289\ \text{K})^4) - 7\ \text{W/m}^2 \cdot \text{K}(T_s - 289\ \text{K}) = 0$$
$$2562\ \text{W/m}^2 - 7.37 \times 10^{-9}\ \text{W/m}^2 \cdot \text{K}^4 T_s^4 - 7\ \text{W/m}^2 \cdot \text{K}\ T_s = 0$$

Solving this equation for T_s using IHT or other software results in T_s = 350 K = 77°C. A temperature of 60°C is typically safe to touch without being burned. The steel sheet would be uncomfortably hot or even cause burning. <

COMMENTS: The individual heat flux terms are

$$\alpha_s G_s = 488\ \text{W/m}^2$$
$$q''_{rad} = \varepsilon\sigma(T_s^4 - T_{sur}^4) = 59\ \text{W/m}^2$$
$$q''_{conv} = h(T_s - T_\infty) = 428\ \text{W/m}^2$$

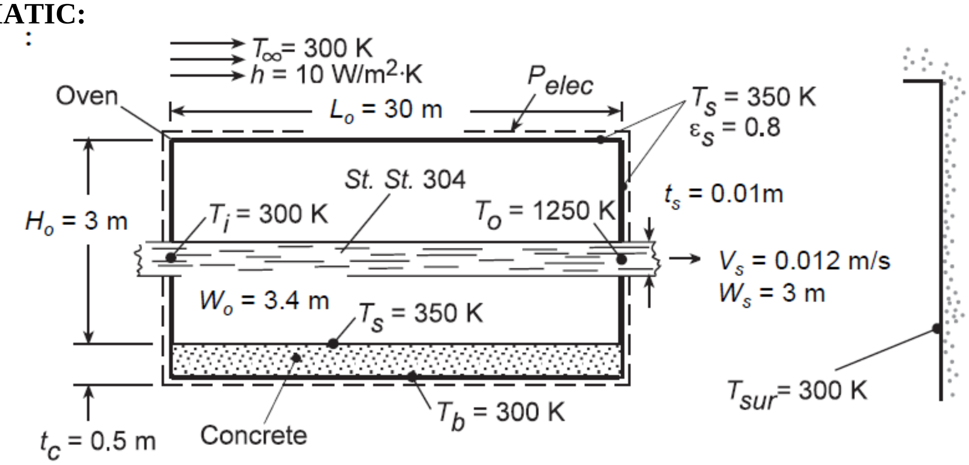
None of these is negligible, although the radiation exchange with the surroundings is smaller than the solar radiation and convection terms.

PROBLEM 1.44

KNOWN: Speed, width, thickness and initial and final temperatures of 304 stainless steel in an annealing process. Dimensions of annealing oven and temperature, emissivity and convection coefficient of surfaces exposed to ambient air and large surroundings of equivalent temperatures. Thickness of pad on which oven rests and pad surface temperatures.

FIND: Oven operating power.

SCHEMATIC:



ASSUMPTIONS: (1) steady-state, (2) Constant properties, (3) Negligible changes in kinetic and potential energy.

PROPERTIES: Table A.1, Stainless 304 $\left(\bar{T} = (T_i + T_o)/2 = 775\,K\right)$: ρ = 7900 kg/m^3, c_p = 578 J/kg·K; Table A.3, Concrete, T = 300 K: k_c = 1.4 W/m·K.

ANALYSIS: The rate of energy addition to the oven must balance the rate of energy transfer to the steel sheet and the rate of heat loss from the oven. Viewing the oven as an open system, Equation (1.12e) yields

$$P_{elec} - q = \dot{m}c_p(T_o - T_i)$$

where q is the heat transferred from the oven. With $\dot{m} = \rho V_s\left(W_s t_s\right)$ and

$$q = \left(2H_oL_o + 2H_oW_o + W_oL_o\right)\times\left[h\left(T_s - T_\infty\right) + \varepsilon_s\sigma\left(T_s^4 - T_{sur}^4\right)\right] + k_c\left(W_oL_o\right)\left(T_s - T_b\right)/t_c\,,$$

it follows that

$$P_{elec} = \rho V_s\left(W_s t_s\right)c_p\left(T_o - T_i\right) + \left(2H_oL_o + 2H_oW_o + W_oL_o\right)\times$$
$$\left[h\left(T_s - T_\infty\right) + \varepsilon_s\sigma\left(T_s^4 - T_{sur}^4\right)\right] + k_c\left(W_oL_o\right)\left(T_s - T_b\right)/t_c$$

$$P_{elec} = 7900\,\text{kg/m}^3 \times 0.012\,\text{m/s}\left(3\,\text{m}\times 0.01\,\text{m}\right)578\,\text{J/kg}\cdot\text{K}\left(1250 - 300\right)\text{K}$$
$$+\left(2\times 3\text{m}\times 30\text{m} + 2\times 3\text{m}\times 3.4\text{m} + 3.4\text{m}\times 30\text{m}\right)[10\,\text{W/m}^2\cdot\text{K}\left(350 - 300\right)\text{K}$$
$$+0.8\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4\left(350^4 - 300^4\right)\text{K}^4] + 1.4\,\text{W/m}\cdot\text{K}\left(3.4\text{m}\times 30\text{m}\right)\left(350 - 300\right)\text{K/0.5m}$$

$$P_{elec} = 1{,}560{,}000\,\text{W} + 302.4\,\text{m}^2\left(500 + 313\right)\text{W/m}^2 + 14{,}280\,\text{W}$$

$$= \left(1{,}560{,}000 + 151{,}200 + 94{,}730 + 14{,}280\right)\text{W} = 1822\ \text{kW}$$ <

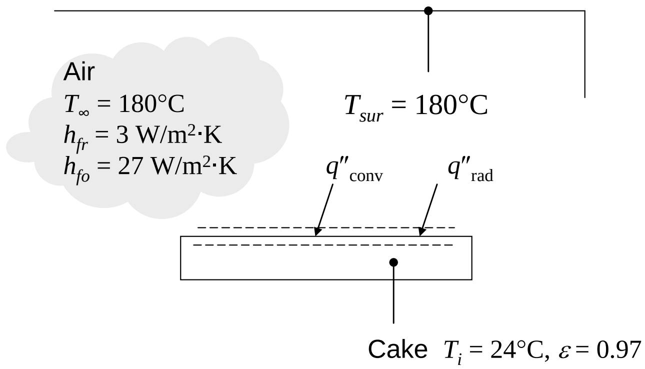
COMMENTS: Of the total energy input, 86% is transferred to the steel while approximately 10%, 5% and 1% are lost by convection, radiation and conduction from the oven. The convection and radiation losses can both be reduced by adding insulation to the side and top surfaces, which would reduce the corresponding value of T_s.

PROBLEM 1.45

KNOWN: Temperatures of small cake as well as oven air and walls. Convection heat transfer coefficient under free and forced convection conditions. Emissivity of cake batter and pan.

FIND: Heat flux to cake under free and forced convection conditions.

SCHEMATIC:



ASSUMPTIONS: (1) Large surroundings.

ANALYSIS: The heat flux to the cake pan and batter is due to convection and radiation. With the surface temperature equal to T_i, when the convection feature is disabled,

$$q''_{fr} = (q''_{\text{conv}} + q''_{\text{rad}}) = h_{fr}(T_\infty - T_i) + \varepsilon\sigma(T_{\text{sur}}^4 - T_i^4)$$
$$= 3\ \text{W/m}^2 \cdot \text{K}(180°\text{C} - 24°\text{C}) + 0.97 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4 \left((180 + 273\ \text{K})^4 - (24 + 273\ \text{K})^4\right) \quad <$$
$$= 470\ \text{W/m}^2 + 1890\ \text{W/m}^2 = 2360\ \text{W/m}^2$$

When the convection feature is activated, the heat flux is

$$q''_{fo} = (q''_{\text{conv}} + q''_{\text{rad}}) = h_{fo}(T_\infty - T_i) + \varepsilon\sigma(T_{\text{sur}}^4 - T_i^4)$$
$$= 27\ \text{W/m}^2 \cdot \text{K}(180°\text{C} - 24°\text{C}) + 0.97 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4 \left((180 + 273\ \text{K})^4 - (24 + 273\ \text{K})^4\right) \quad <$$
$$= 4210\ \text{W/m}^2 + 1890\ \text{W/m}^2 = 6100\ \text{W/m}^2$$

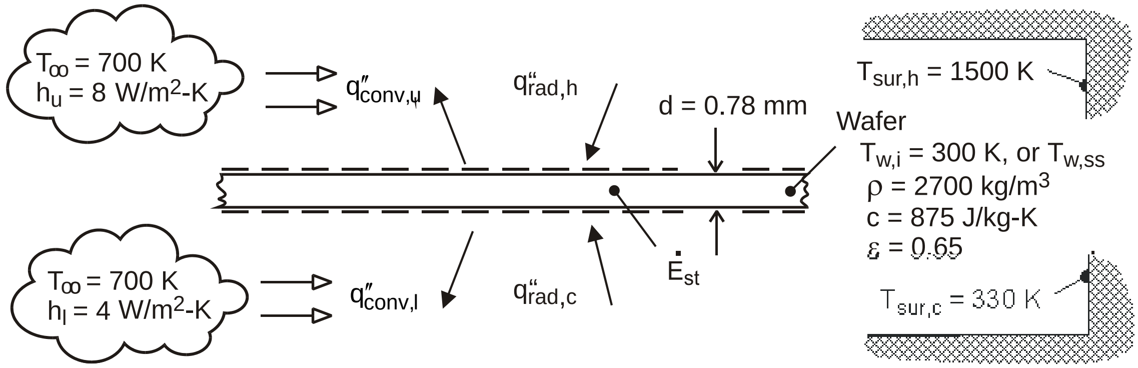
COMMENTS: Under free convection conditions, the convection contribution is about 20% of the total heat flux. When forced convection is activated, convection becomes larger than radiation, accounting for 69% of the total heat flux. The cake will bake faster under forced convection conditions.

PROBLEM 1.46

KNOWN: Silicon wafer positioned in furnace with top and bottom surfaces exposed to hot and cool zones, respectively.

FIND: (a) Initial rate of change of the wafer temperature corresponding to the wafer temperature $T_{w,i} = 300\,K$, and (b) Steady-state temperature reached if the wafer remains in this position. How significant is convection for this situation? Sketch how you'd expect the wafer temperature to vary as a function of vertical distance.

SCHEMATIC:



ASSUMPTIONS: (1) Wafer temperature is uniform, (2) Transient conditions when wafer is initially positioned, (3) Hot and cool zones have uniform temperatures, (3) Radiation exchange is between small surface (wafer) and large enclosure (chamber, hot or cold zone), and (4) Negligible heat losses from wafer to mounting pin holder.

ANALYSIS: The energy balance on the wafer illustrated in the schematic above includes convection from the upper (u) and lower (l) surfaces with the ambient gas, radiation exchange with the hot- and cool-zone (chamber) surroundings, and the rate of energy storage term for the transient condition.

$$\dot{E}''_{in} - \dot{E}''_{out} = \dot{E}''_{st}$$

$$q''_{rad,h} + q''_{rad,c} - q''_{conv,u} - q''_{conv,l} = \rho c d \frac{dT_w}{dt}$$

$$\varepsilon\sigma\left(T_{sur,h}^4 - T_w^4\right) + \varepsilon\sigma\left(T_{sur,c}^4 - T_w^4\right) - h_u\left(T_w - T_\infty\right) - h_l\left(T_w - T_\infty\right) = \rho c d \frac{dT_w}{dt}$$

(a) For the initial condition, the time rate of temperature change of the wafer is determined using the energy balance above with $T_w = T_{w,i} = 300\,K$,

$$0.65 \times 5.67 \times 10^{-8}\,W/m^2 \cdot K^4\left(1500^4 - 300^4\right)K^4 + 0.65 \times 5.67 \times 10^{-8}\,W/m^2 \cdot K^4\left(330^4 - 300^4\right)K^4$$

$$-8\,W/m^2 \cdot K\left(300 - 700\right)K - 4\,W/m^2 \cdot K\left(300 - 700\right)K =$$

$$2700\,kg/m^3 \times 875\,J/kg \cdot K \times 0.00078\,m\left(dT_w/dt\right)_i$$

$$\left(dT_w/dt\right)_i = 104\,K/s$$ <

(b) For the steady-state condition, the energy storage term is zero, and the energy balance can be solved for the steady-state wafer temperature, $T_w = T_{w,ss}$.

Continued …..

PROBLEM 1.46 (Cont.)

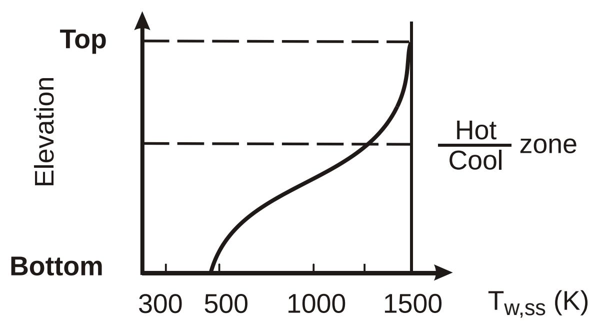
$$0.65\sigma\left(1500^4 - T_{w,ss}^4\right)K^4 + 0.65\sigma\left(330^4 - T_{w,ss}^4\right)K^4$$

$$-8\,W/m^2\cdot K\left(T_{w,ss} - 700\right)K - 4\,W/m^2\cdot K\left(T_{w,ss} - 700\right)K = 0$$

$$T_{w,ss} = 1251\,K$$ <

To determine the relative importance of the convection processes, re-solve the energy balance above ignoring those processes to find $\left(dT_w / dt\right)_i = 101\ K/s$ and $T_{w,ss} = 1262\ K$. We conclude that the radiation exchange processes control the initial time rate of temperature change and the steady-state temperature.

If the wafer were elevated above the present operating position, its temperature would increase, since the lower surface would begin to experience radiant exchange with progressively more of the hot zone chamber. Conversely, by lowering the wafer, the upper surface would experience less radiant exchange with the hot zone chamber, and its temperature would decrease. The temperature-distance trend might appear as shown in the sketch.

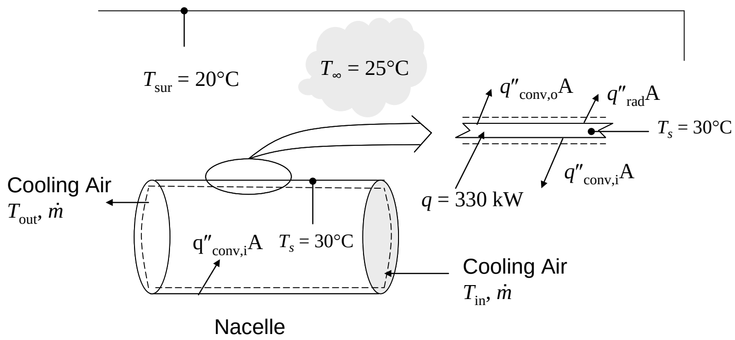


PROBLEM 1.47

KNOWN: Total rate of heat transfer leaving nacelle (from Example 1.3). Dimensions and emissivity of the nacelle, ambient and surrounding temperatures, convection heat transfer coefficient exterior to nacelle. Temperature of exiting forced air flow.

FIND: Required mass flow rate of forced air flow.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Large surroundings, (3) Surface of the nacelle that is adjacent to the hub is adiabatic, (4) Forced air exits nacelle at the nacelle surface temperature.

ANALYSIS: The total rate of heat transfer leaving the nacelle is known from Example 1.3 to be $q = 0.33 \times 10^6$ W = 330 kW. Heat is removed from the nacelle by radiation and convection from the exterior surface of the nacelle (q_{rad} and $q_{conv,o}$, respectively), and by convection from the interior surface to the forced flow of air through the nacelle ($q_{conv,i}$). An energy balance on the nacelle based upon the upper-right part of the schematic yields

$$q = q_{rad} + q_{conv,o} + q_{conv,i} = A\left[q''_{rad} + q''_{conv,o}\right] + q_{conv,i}$$

Thus the required rate of heat removal by the forced air is given by

$$q_{conv,i} = q - A\left[q''_{rad} + q''_{conv,o}\right] = q - \left[\pi DL + \frac{\pi D^2}{4}\right]\left[\varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) + h\left(T_s - T_\infty\right)\right]$$

In order to maintain a nacelle surface temperature of T_s = 30°C, the required $q_{conv,i}$ is

$$\begin{aligned} q_{conv,i} &= 330 \text{ kW} - \left[\pi \times 3 \text{ m} \times 6 \text{ m} + \frac{\pi \times (3 \text{ m})^2}{4}\right] \times \\ &\left[0.83 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 \left((273+30)^4 - (273+20)^4\right)\text{K}^4 + 35 \text{ W/m}^2 \cdot \text{K}(30-25)\text{K}\right] \\ &= 330 \text{ kW} - (3 \text{ kW} + 11 \text{ kW}) = 316 \text{ kW} \end{aligned}$$

The required mass flow rate of air can be found by applying an energy balance to the air flowing through the nacelle, as shown by the control volume on the lower left of the schematic. From Equation 1.12e:

$$\dot{m} = \frac{q_{conv,i}}{c_p(T_{out} - T_{in})} = \frac{q_{conv,i}}{c_p(T_s - T_\infty)} = \frac{316 \text{ kW}}{1007 \text{ J/kg} \cdot \text{K}(30-25)\text{K}} = 63 \text{ kg/s} \qquad <$$

Continued...

PROBLEM 1.47 (Cont.)

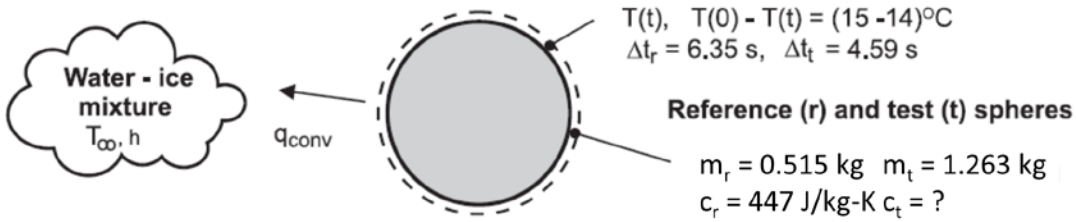
COMMENTS: (1) With the surface temperature lowered to 30°C, the heat lost by radiation and convection from the exterior surface of the nacelle is small, and most of the heat must be removed by convection to the interior forced air flow. (2) The air mass flow rate corresponds to a velocity of around $V = \dot{m} / \rho A_c = \dot{m} / (\rho \pi D^2 / 4)$ = 7 m/s, using an air density of 1.1 kg/m^3 and assuming that the air flows through the entire nacelle cross-sectional area. This would lead to uncomfortable working conditions unless the forced air flow were segregated from the working space. (3) The required heat transfer coefficient on the interior surface can be estimated as $h_i = q_{\text{conv,i}}/(\pi DL(T_s - T_\infty))$ = 1100 W/m^2·K. In Chapter 8, you will learn whether this heat transfer coefficient can be achieved under the given conditions.

PROBLEM 1.48

KNOWN: Elapsed times corresponding to a temperature change from 15 to 14°C for a reference sphere and test sphere of unknown composition suddenly immersed in a stirred water-ice mixture. Mass and specific heat of reference sphere.

FIND: Specific heat of the test sphere of known mass.

SCHEMATIC:



ASSUMPTIONS: (1) Spheres are of equal diameter, (2) Spheres experience temperature change from 15 to 14°C, (3) Spheres experience same convection heat transfer rate when the time rates of surface temperature are observed, (4) At any time, the temperatures of the spheres are uniform, (5) Negligible heat loss through the thermocouple wires.

PROPERTIES: Reference-grade sphere material: c_r = 447 J/kg K.

ANALYSIS: Apply the conservation of energy requirement at an instant of time, Equation 1.12c, after a sphere has been immersed in the ice-water mixture at T_∞.

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$$

$$-q_{conv} = mc\frac{dT}{dt}$$

where $q_{conv} = h\,A_s\left(T - T_\infty\right)$. Since the temperatures of the spheres are uniform, the change in energy storage term can be represented with the time rate of temperature change, dT/dt. The convection heat rates are equal at this instant of time, and hence the change in energy storage terms for the reference (r) and test (t) spheres must be equal.

$$m_r\ c_r\ \left.\frac{dT}{dt}\right)_r = m_t\ c_t\ \left.\frac{dT}{dt}\right)_t$$

Approximating the instantaneous differential change, dT/dt, by the difference change over a short period of time, ΔT/Δt, the specific heat of the test sphere can be calculated.

$$0.515\ \text{kg} \times 447\,\text{J}/\text{kg}\cdot\text{K}\ \frac{(15-14)\text{K}}{6.35\,\text{s}} = 1.263\,\text{kg} \times c_t \times \frac{(15-14)\text{K}}{4.59\,\text{s}}$$

$$c_t = 132\ \text{J}/\text{kg}\cdot\text{K}$$ <

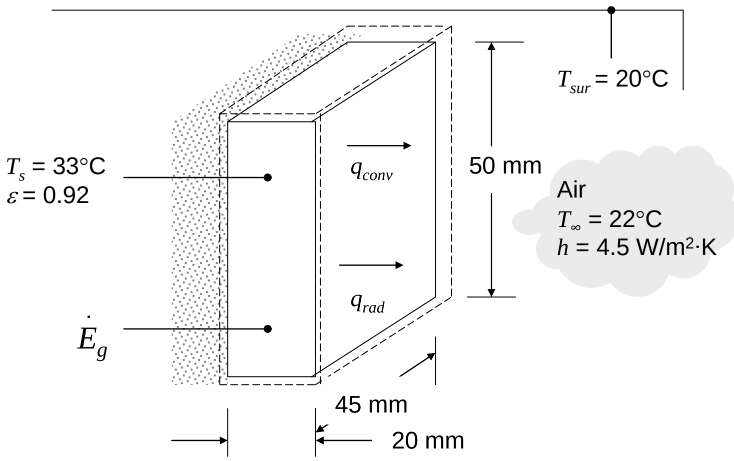
COMMENTS: Why was it important to perform the experiments with the reference and test spheres over the same temperature range (from 15 to 14°C)? Why does the analysis require that the spheres have uniform temperatures at all times?

PROBLEM 1.49

KNOWN: Dimensions and emissivity of a cell phone charger. Surface temperature when plugged in. Temperature of air and surroundings. Convection heat transfer coefficient. Cost of electricity.

FIND: Daily cost of leaving the charger plugged in when not in use.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Convection and radiation are from five exposed surfaces of charger, (3) Large surroundings, (4) Negligible heat transfer from back of charger to wall and outlet.

ANALYSIS: At steady-state, an energy balance on the charger gives $\dot{E}_{\text{in}} + \dot{E}_g = 0$, where $\dot{E}_g$ represents the conversion from electrical to thermal energy. The exposed area is A = (50 mm × 45 mm) + 2(50 mm × 20 mm) + 2(45 mm × 20 mm) = 6050 mm^2. Thus,

$$\dot{E}_g = (q_{\text{conv}} + q_{\text{rad}}) = hA(T_s - T_\infty) + \varepsilon\sigma A(T_s^4 - T_{\text{sur}}^4)$$

$$= \left[4.5\ \text{W/m}^2\cdot\text{K}(33°\text{C} - 22°\text{C}) + 0.92 \times 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\left((306\ \text{K})^4 - (293\ \text{K})^4\right)\right] \times 6050 \times 10^{-6}\ \text{m}^2$$

$$= 0.74\ \text{W}$$

This is the total rate of electricity used while the charger is plugged in. The daily cost of electricity is

Cost = 0.74 W × \$0.18/kW·h × 1 kW/1000 W × 24 h/day = \$0.0032/day <

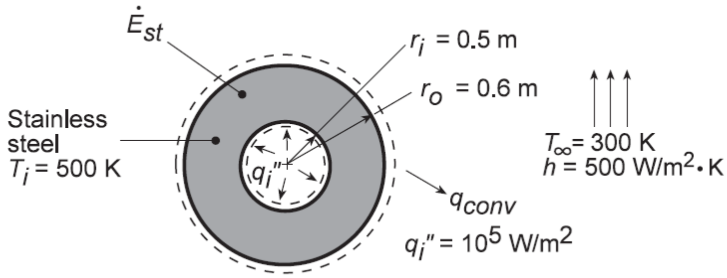
COMMENTS: (1) The radiation and convection heat fluxes are 73 W/m^2 and 50 W/m^2, respectively. Therefore, both modes of heat transfer are important. (2) The cost of leaving the charger plugged in when not in use is small.

PROBLEM 1.50

KNOWN: Inner surface heating and new environmental conditions associated with a spherical shell of prescribed dimensions and material.

FIND: (a) Governing equation for variation of wall temperature with time. Initial rate of temperature change, (b) Steady-state wall temperature, (c) Effect of convection coefficient on canister temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible temperature gradients in wall, (2) Constant properties, (3) Uniform, time-independent heat flux at inner surface.

PROPERTIES: *Table A.1*, Stainless Steel, AISI 302: $\rho = 8055\ \text{kg/m}^3$, $c_p = 535$ J/kg·K.

ANALYSIS: (a) Performing an energy balance on the shell at an instant of time, $\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$. Identifying relevant processes and solving for dT/dt,

$$q_i''\left(4\pi r_i^2\right) - h\left(4\pi r_o^2\right)\left(T - T_\infty\right) = \rho \frac{4}{3}\pi\left(r_o^3 - r_i^3\right)c_p \frac{dT}{dt}$$

$$\frac{dT}{dt} = \frac{3}{\rho c_p\left(r_o^3 - r_i^3\right)}\left[q_i'' r_i^2 - h r_o^2\left(T - T_\infty\right)\right].$$

Substituting numerical values for the initial condition, find

$$\left.\frac{dT}{dt}\right)_i = \frac{3\left[10^5 \frac{W}{m^2}\left(0.5m\right)^2 - 500\frac{W}{m^2 \cdot K}\left(0.6m\right)^2\left(500 - 300\right)K\right]}{8055\frac{kg}{m^3}510\frac{J}{kg \cdot K}\left[\left(0.6\right)^3 - \left(0.5\right)^3\right]m^3}$$

$$\left.\frac{dT}{dt}\right)_i = -0.084\,K/s.$$ <

(b) Under steady-state conditions with $\dot{E}_{st} = 0$, it follows that

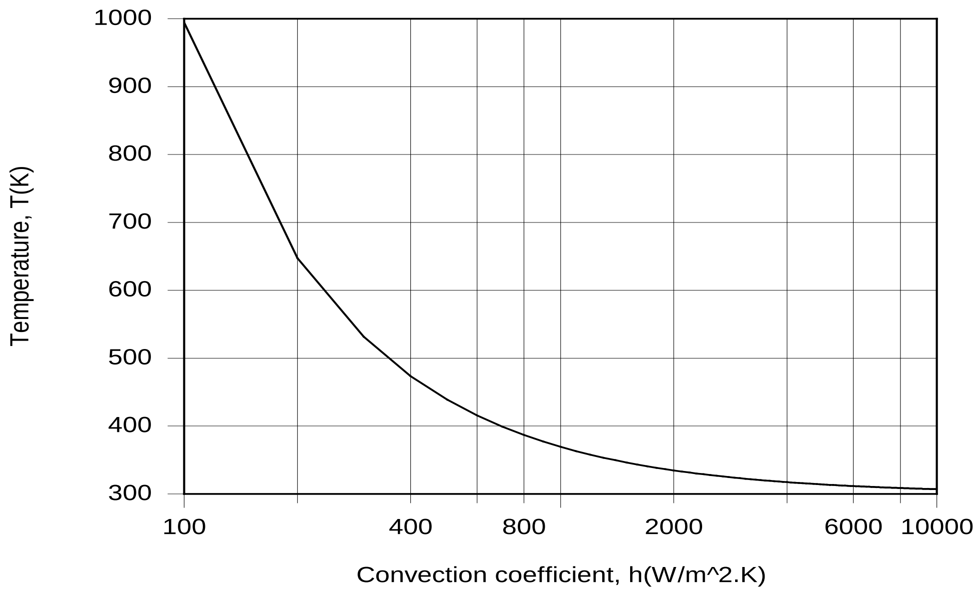
$$q_i''\left(4\pi r_i^2\right) = h\left(4\pi r_o^2\right)\left(T - T_\infty\right)$$

$$T = T_\infty + \frac{q_i''}{h}\left(\frac{r_i}{r_o}\right)^2 = 300K + \frac{10^5\,W/m^2}{500W/m^2 \cdot K}\left(\frac{0.5m}{0.6m}\right)^2 = 439K$$ <

Continued …..

PROBLEM 1.50 (Cont.)

(c) Parametric calculations were performed using the IHT *First Law Model* for an *Isothermal Hollow Sphere*. As shown below, there is a sharp increase in temperature with decreasing values of h < 1000 $W/m^2 \cdot K$. For T > 380 K, boiling will occur at the canister surface, and for T > 410 K a condition known as film boiling (Chapter 10) will occur. The condition corresponds to a precipitous reduction in h and increase in T.



Although the canister remains well below the melting point of stainless steel for h = 100 $W/m^2 \cdot K$, boiling should be avoided, in which case the convection coefficient should be maintained at h > 1000 $W/m^2 \cdot K$.

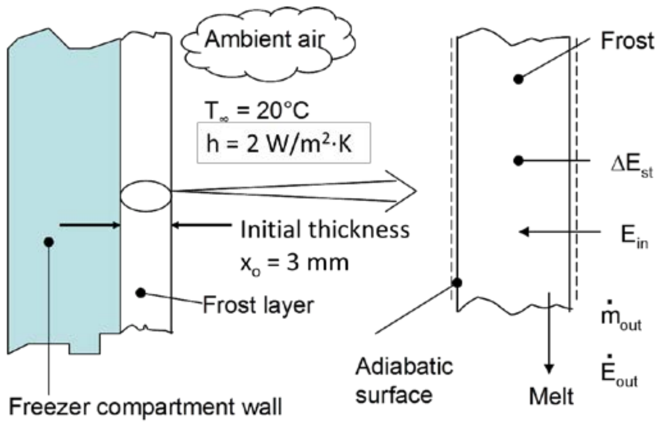
COMMENTS: The governing equation of part (a) is a first order, nonhomogenous differential equation with constant coefficients. Its solution is $\theta = (S/R)\left(1 - e^{-Rt}\right) + \theta_i e^{-Rt}$, where $\theta \equiv T - T_\infty$, $S \equiv 3q_i'' r_i^2 / \rho c_p \left(r_o^3 - r_i^3\right)$, $R = 3hr_o^2 / \rho c_p \left(r_o^3 - r_i^3\right)$. Note results for $t \rightarrow \infty$ and for S = 0.

PROBLEM 1.51

KNOWN: Frost formation of 3-mm thickness on a freezer compartment. Surface exposed to convection process with ambient air.

FIND: Time required for the frost to melt, t_m.

SCHEMATIC:



ASSUMPTIONS: (1) Frost is isothermal at the fusion temperature, T_f, (2) The water melt falls away from the exposed surface, (3) Frost exchanges radiation with surrounding frost, so net radiation exchange is negligible, and (4) Backside surface of frost formation is adiabatic.

PROPERTIES: Frost, $\rho_f = 770\,\text{kg/m}^3$, $h_{sf} = 334\,\text{kJ/kg}$.

ANALYSIS: The time t_m required to melt a 3-mm thick frost layer may be determined by applying a mass balance and an energy balance (Eq 1.12b) over the differential time interval dt to a control volume around the frost layer.

$$dm_{st} = -\dot{m}_{out}dt \qquad dE_{st} = \left(\dot{E}_{in} - \dot{E}_{out}\right)dt \qquad (1a,b)$$

With h_f as the enthalpy of the melt and h_s as the enthalpy of frost, we have

$$dE_{st} = dm_{st}h_s \qquad \dot{E}_{out}dt = \dot{m}_{out}h_f\,dt \qquad (2a,b)$$

Combining Eqs. (1a) and (2a,b), Eq. (1b) becomes (with $h_{sf} = h_f - h_S$)

$$\dot{m}_{out}h_{sf}\,dt = \dot{E}_{in}dt = q''_{conv}A_s dt$$

Integrating both sides of the equation with respect to time, find

$$\rho_f A_s h_{sf} x_o = h A_s \left(T_\infty - T_f\right)t_m$$

$$t_m = \frac{\rho_f h_{sf} x_o}{h\left(T_\infty - T_f\right)}$$

$$t_m = \frac{700\,\text{kg/m}^3 \times 334\times10^3\,\text{J/kg}\times 0.003\text{m}}{2\,\text{W/m}^2\cdot\text{K}\left(20-0\right)\text{K}} = 17{,}540\ \text{s} = 4.9\ \text{h}$$

<

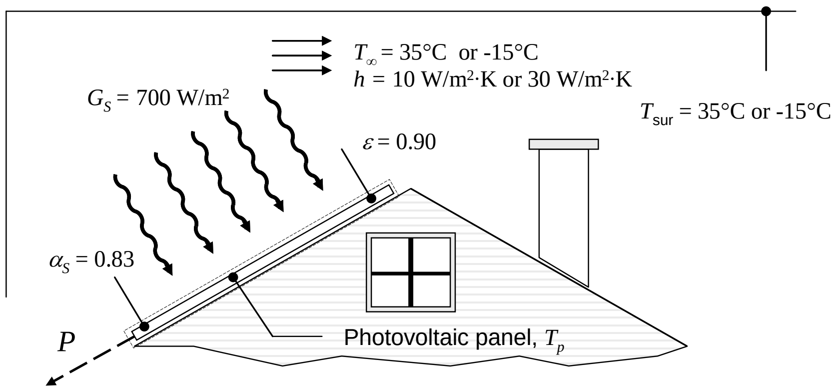
COMMENTS: (1) The energy balance could be formulated intuitively by recognizing that the total heat *in* by convection during the time interval $t_m\left(q''_{conv}\cdot t_m\right)$ must be equal to the total latent energy for melting the frost layer $\left(\rho\, x_o h_{sf}\right)$. This equality is directly comparable to the derived expression above for t_m.

PROBLEM 1.52

KNOWN: Dimensions, emissivity, and solar absorptivity of solar photovoltaic panel. Solar irradiation, air and surroundings temperature, and convection coefficient. Expression for conversion efficiency.

FIND: Electrical power output on (a) a still summer day, and (b) a breezy winter day.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Lower surface of solar panel is insulated, (3) Radiation from the environment can be treated as radiation from large surroundings, with $\alpha = \varepsilon$.

ANALYSIS: Recognize that there is conversion from thermal to electrical energy, therefore there is a negative generation term equal to the electrical power. Performing an energy balance on the solar panel gives

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \dot{E}_g = 0$$

$$q_{\text{rad}} - q_{\text{conv}} - P = 0$$

$$\left[\alpha_S G_S - \varepsilon\sigma(T_s^4 - T_{\text{sur}}^4)\right]A - hA(T_s - T_\infty) - \eta\alpha_S G_S A = 0$$

Dividing by A, and substituting the expression for η as a function of T_p yields

$$\left[\alpha_S G_S - \varepsilon\sigma(T_p^4 - T_{\text{sur}}^4)\right] - h(T_p - T_\infty) - (0.553 - 0.001T_p)\alpha_S G_S = 0$$

(a) Substituting the parameter values for a summer day:

$$0.83\times 700\text{ W/m}^2 - 0.90\times 5.67\times 10^{-8}\text{ W/m}^2\cdot\text{K}^4(T_p^4 - (308\text{ K})^4) - 10\text{ W/m}^2\cdot\text{K}(T_p - 308\text{ K})$$

$$-(0.553 - 0.001T_p)\times 0.83\times 700\text{ W/m}^2 = 0$$

$$3799\text{ W/m}^2 - 5.1\times 10^{-8}\text{ W/m}^2\cdot\text{K}^4 T_p^4 - 9.42\text{ W/m}^2\cdot\text{K}\ T_p = 0$$

Solving this equation for T_p using IHT or other software results in T_p = 335 K. The electrical power can then be found from

$$P = \eta\alpha_S G_S A = (0.553 - 0.001T_p)\alpha_S G_S A$$

$$= (0.553 - 0.001\text{ K}^{-1}\times 335\text{ K})\times 0.83\times 700\text{ W/m}^2\times 8\text{ m}^2 = 1010\text{ W}$$ <

(b) Repeating the calculation for the winter conditions yields T_p = 270 K, P = 1310 W. <

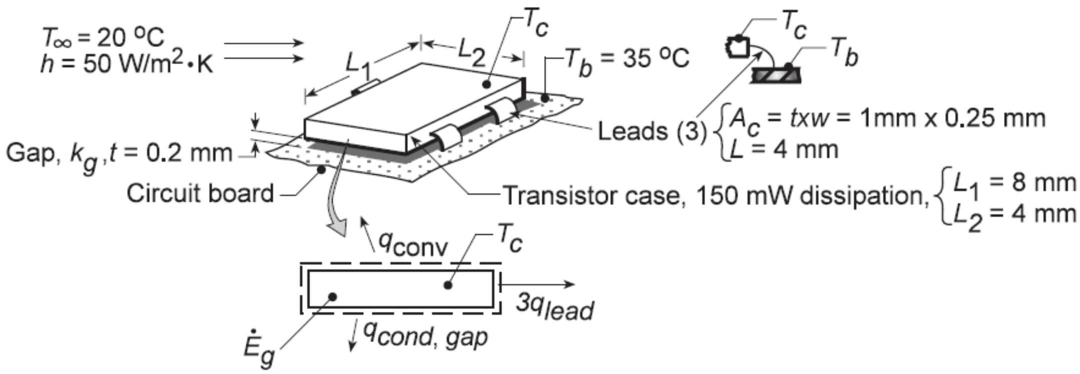
COMMENTS: (1) The conversion efficiency for most photovoltaic materials is higher at lower temperatures. Therefore, for the same solar irradiation, more electrical power is generated in winter conditions. (2) The total solar energy generated would generally be less in the winter due to lower irradiation values and a shorter day.

PROBLEM 1.53

KNOWN: Surface-mount transistor with prescribed dissipation and convection cooling conditions.

FIND: (a) Case temperature for mounting arrangement with air-gap and conductive paste between case and circuit board, (b) Consider options for increasing $\dot{E}_g$, subject to the constraint that T_c = 40°C.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Transistor case is isothermal, (3) Upper surface experiences convection; negligible losses from edges, (4) Leads provide conduction path between case and board, (5) Negligible radiation, (6) Negligible energy generation in leads due to current flow, (7) Negligible convection from surface of leads.

PROPERTIES: (Given): Air, $k_{g,a}$ = 0.0263 W/m·K; Paste, $k_{g,p}$ = 0.12 W/m·K; Metal leads, k_ℓ = 25 W/m·K.

ANALYSIS: (a) Define the transistor as the system and identify modes of heat transfer.

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \Delta\dot{E}_{st} = 0$$

$$-q_{conv} - q_{cond,gap} - 3q_{lead} + \dot{E}_g = 0$$

$$-hA_s\left(T_c - T_\infty\right) - k_g A_s \frac{T_c - T_b}{t} - 3k_\ell A_c \frac{T_c - T_b}{L} + \dot{E}_g = 0$$

where $A_s = L_1 \times L_2$ = 4 × 8 mm² = 32 × 10⁻⁶ m² and A_c = t × w = 0.25 × 1 mm² = 25 × 10⁻⁸ m².

Rearranging and solving for T_c,

$$T_c = \left\{hA_sT_\infty + \left[k_gA_s/t + 3\left(k_\ell A_c/L\right)\right]T_b + \dot{E}_g\right\} / \left[hA_s + k_gA_s/t + 3\left(k_\ell A_c/L\right)\right]$$

Substituting numerical values, with the *air-gap condition* ($k_{g,a}$ = 0.0263 W/m·K)

$$T_c = \left\{50\,\mathrm{W/m^2\cdot K} \times 32\times10^{-6}\,\mathrm{m^2} \times 20^\circ\mathrm{C} + \left[\left(0.0263\,\mathrm{W/m\cdot K} \times 32\times10^{-6}\,\mathrm{m^2}/0.2\times10^{-3}\,\mathrm{m}\right)\right.\right.$$
$$\left.\left.+3\left(25\,\mathrm{W/m\cdot K}\times 25\times10^{-8}\,\mathrm{m^2}/4\times10^{-3}\,\mathrm{m}\right)\right]35^\circ\mathrm{C}\right\} / \left[1.600\times10^{-3} + 4.208\times10^{-3} + 4.688\times10^{-3}\right]\mathrm{W/K}$$

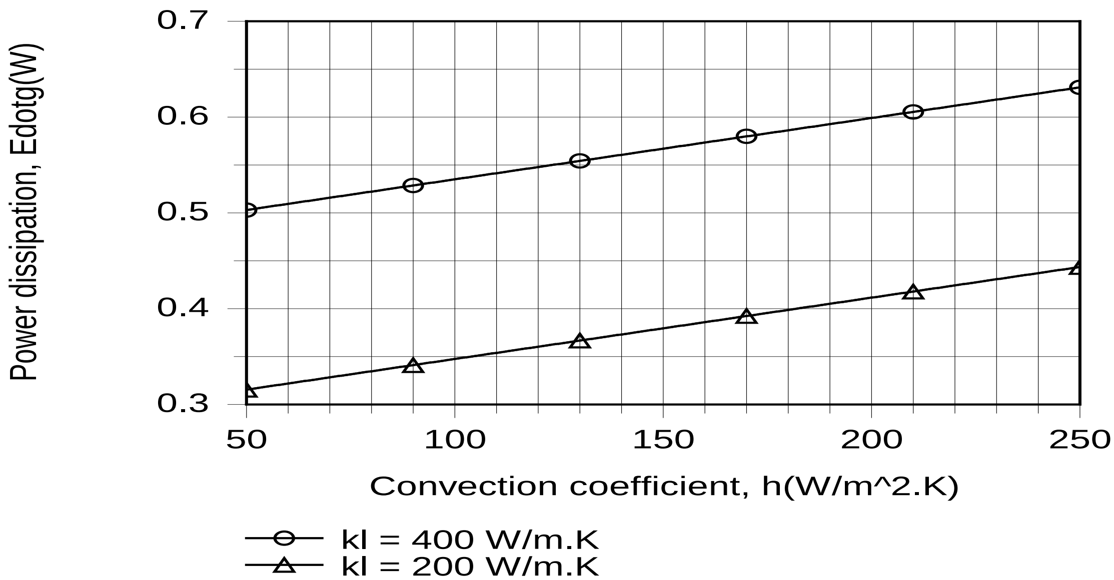
$$T_c = 47.0^\circ\mathrm{C}.$$ <

Continued...

PROBLEM 1.53 (Cont.)

With the *paste condition* ($k_{g,p}$ = 0.12 W/m·K), T_c = 39.9°C. As expected, the effect of the conductive paste is to improve the coupling between the circuit board and the case. Hence, T_c decreases.

(b) Using the keyboard to enter model equations into the workspace, IHT has been used to perform the desired calculations. For values of k_ℓ = 200 and 400 W/m·K and convection coefficients in the range from 50 to 250 W/m^2·K, the energy balance equation may be used to compute the power dissipation for a maximum allowable case temperature of 40°C.



As indicated by the energy balance, the power dissipation increases linearly with increasing h, as well as with increasing k_ℓ. For h = 250 W/m^2·K (enhanced air cooling) and k_ℓ = 400 W/m·K (copper leads), the transistor may dissipate up to 0.63 W.

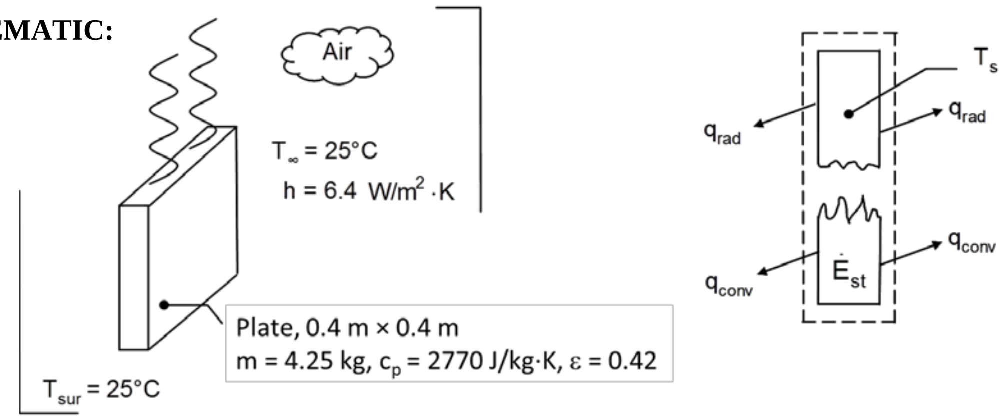
COMMENTS: Additional benefits may be derived by increasing heat transfer across the gap separating the case from the board, perhaps by inserting a highly conductive material in the gap.

PROBLEM 1.54

KNOWN: Hot plate suspended in a room, plate temperature, room temperature and surroundings temperature, convection coefficient and plate emissivity, mass and specific heat of the plate.

FIND: (a) The time rate of change of the plate temperature, and (b) Heat loss by convection and heat loss by radiation.

SCHEMATIC:



ASSUMPTIONS: (1) Plate is isothermal and at uniform temperature, (2) Large surroundings, (3) Negligible heat loss through suspension wires.

ANALYSIS: For a control volume about the plate, the conservation of energy requirement is

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st} \tag{1}$$

where $$\dot{E}_{st} = mc_p \frac{dT}{dt} \tag{2}$$

and $$\dot{E}_{in} - \dot{E}_{out} = \varepsilon A\sigma(T_{sur}^4 - T_s^4) + hA(T_\infty - T_s) \tag{3}$$

Combining Eqs. (1) through (3) yields $\dfrac{dT}{dt} = \dfrac{A[\varepsilon\sigma(T_{sur}^4 - T_s^4) + h(T_\infty - T_s)]}{mc_p}$

Noting that $T_{sur} = 25^oC + 273\text{ K} = 298\text{ K}$ and $T_s = 225^oC + 273\text{ K} = 498\text{ K}$,

$$\frac{dT}{dt} = \frac{2\times0.4\text{ m}\times0.4\text{ m}\left[0.42\times5.67\times10^{-8}\dfrac{\text{W}}{\text{m}^2\cdot\text{K}^4}\times(298^4\text{-}498^4)\text{ K}^4 + 6.4\dfrac{\text{W}}{\text{m}^2\cdot\text{K}}\times(25^oC\text{-}225^oC)\right]}{4.25\text{ kg}\times2770\ \dfrac{\text{J}}{\text{kg}\cdot\text{K}}}$$

$= -0.0695$ K/s <

The heat gain by radiation is the first term in the numerator of the preceding expression and is

$q_{rad} = -409$ W (a loss of 409 W) <

The heat gain by convection is the second term in the preceding expression and is

$q_{conv} = -410$ W (a loss of 410 W) <

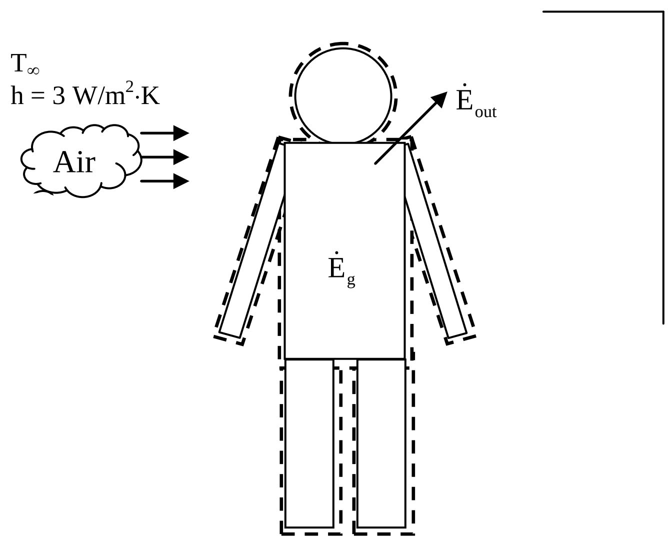
COMMENTS: (1) Note the importance of using kelvins when working with radiation heat transfer. (2) The temperature difference in Newton’s law of cooling may be expressed in either kelvins or degrees Celsius. (3) Radiation and convection losses are of the same magnitude. This is typical of many natural convection systems involving gases such as air.

PROBLEM 1.55

KNOWN: Daily thermal energy generation, surface area, temperature of the environment, and heat transfer coefficient.

FIND: (a) Skin temperature when the temperature of the environment is 20ºC, and (b) Rate of perspiration to maintain skin temperature of 33ºC when the temperature of the environment is 33ºC.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Thermal energy is generated at a constant rate throughout the day, (3) Air and surrounding walls are at same temperature, (4) Skin temperature is uniform, (5) Bathing suit has no effect on heat loss from body, (6) Heat loss is by convection and radiation to the environment, and by perspiration in Part 2. (Heat loss due to respiration, excretion of waste, etc., is negligible.), (7) Large surroundings.

PROPERTIES: Table A.11, skin: $\varepsilon = 0.95$, Table A.6, water (306 K): $\rho = 994\ kg/m^3$, $h_{fg} = 2421$ kJ/kg.

ANALYSIS:
(a) The rate of energy generation is:

$$\dot{E}_g = 2000 \times 10^3\ \text{cal/day} \big/ (0.239\ \text{cal/J} \times 86{,}400\ \text{s/day}) = 96.9\ \text{W}$$

Under steady-state conditions, an energy balance on the human body yields:

$$\dot{E}_g - \dot{E}_{out} = 0$$

Thus $\dot{E}_{out} = q = 96.9$ W. Energy outflow is due to convection and net radiation from the surface to the environment, Equations 1.3a and 1.7, respectively.

$$\dot{E}_{out} = hA(T_s - T_\infty) + \varepsilon\sigma A(T_s^4 - T_{sur}^4)$$

Substituting numerical values

Continued…

PROBLEM 1.55 (Cont.)

$$96.9\text{ W} = 3\text{ W/m}^2\cdot\text{K}\times1.8\text{ m}^2\times(T_s - 293\text{ K})$$
$$+\ 0.95\times5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\times1.8\text{ m}^2\times(T_s^4 - (293\text{ K})^4)$$

and solving either by trial-and-error or using *IHT* or other equation solver, we obtain

$$T_s = 299\text{ K} = 26^\circ\text{C} \qquad <$$

Since the comfortable range of skin temperature is typically 32 – 35ºC, we usually wear clothing warmer than a bathing suit when the temperature of the environment is 20ºC.

(b) If the skin temperature is 33ºC when the temperature of the environment is 33ºC, there will be no heat loss due to convection or radiation. Thus, all the energy generated must be removed due to perspiration:

$$\dot{E}_{out} = \dot{m}h_{fg}$$

We find:

$$\dot{m} = \dot{E}_{out}/h_{fg} = 96.9\text{ W}/2421\text{ kJ/kg} = 4.0\times10^{-5}\text{ kg/s}$$

This is the perspiration rate in mass per unit time. The volumetric rate is:

$$\dot{\forall} = \dot{m}/\rho = 4.0\times10^{-5}\text{ kg/s} / 994\text{ kg/m}^3 = 4.0\times10^{-8}\text{ m}^3\text{/s} = 4.0\times10^{-5}\ \ell\text{/s} \qquad <$$

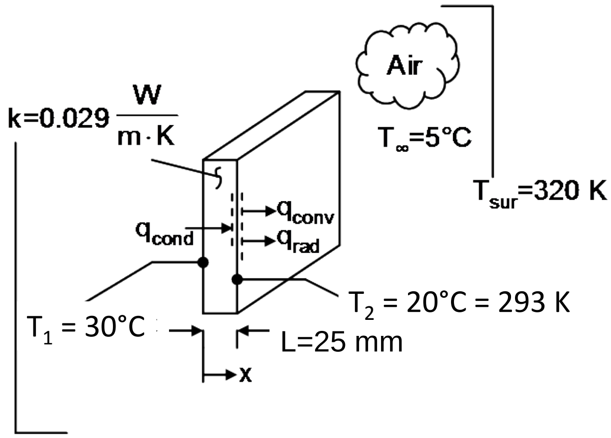
COMMENTS: (1) In Part 1, heat losses due to convection and radiation are 32.4 W and 60.4 W, respectively. Thus, it would not have been reasonable to neglect radiation. Care must be taken to include radiation when the heat transfer coefficient is small, even if the problem statement does not give any indication of its importance. (2) The rate of thermal energy generation is not constant throughout the day; it adjusts to maintain a constant core temperature. Thus, the energy generation rate may decrease when the temperature of the environment goes up, or increase (for example, by shivering) when the temperature of the environment is low. (3) The skin temperature is not uniform over the entire body. For example, the extremities are usually cooler. Skin temperature also adjusts in response to changes in the environment. As the temperature of the environment increases, more blood flow will be directed near the surface of the skin to increase its temperature, thereby increasing heat loss. (4) If the perspiration rate found in Part 2 was maintained for eight hours, the person would lose 1.2 liters of liquid. This demonstrates the importance of consuming sufficient amounts of liquid in warm weather.

PROBLEM 1.56

KNOWN: Thermal conductivity, thickness and temperature difference across a sheet of rigid extruded insulation. Cold wall temperature, surroundings temperature, ambient temperature and emissivity.

FIND: (a) The value of the convection heat transfer coefficient on the cold wall side in units of $W/m^2\cdot°C$ or $W/m^2\cdot K$, and, (b) The cold wall surface temperature for emissivities over the range $0.05 \le \varepsilon \le 0.95$ for a hot wall temperature of T_1 = 30 °C.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional conduction in the x-direction, (2) Steady-state conditions, (c) Constant properties, (4) Large surroundings.

ANALYSIS:
(a) An energy balance on the control surface shown in the schematic yields

$$\dot{E}_{in} = \dot{E}_{out} \quad \text{or} \quad q_{cond} = q_{conv} + q_{rad}$$

Substituting from Fourier's law, Newton's law of cooling, and Eq. 1.7 yields

$$k\,\frac{T_1 - T_2}{L} = h(T_2 - T_\infty) + \varepsilon\sigma(T_2^4 - T_{sur}^4) \qquad (1)$$

or $$h = \frac{1}{(T_2 - T_\infty)}\left[k\frac{T_1 - T_2}{L} - \varepsilon\sigma(T_2^4 - T_{sur}^4)\right]$$

Substituting values,

$$h = \frac{1}{(20 - 5)^{o}C}\left[0.029\frac{W}{m\cdot K} \times \frac{(30 - 20)^{o}C}{0.02\ m} - 0.95 \times 5.67 \times 10^{-8}\frac{W}{m^2\cdot K^4}(293^4 - 320^4)\ K^4\right]$$

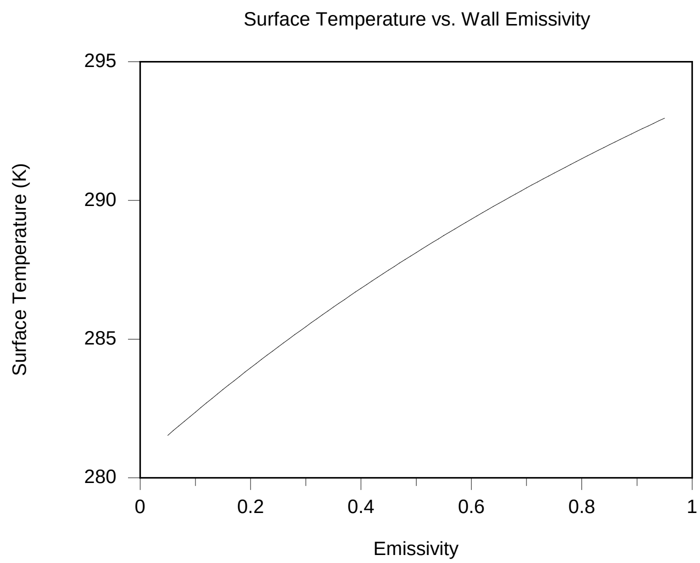
$$h = 12.2\frac{W}{m^2\cdot K}$$ <

Since temperature differences of 1 K and 1°C are equal, $h = 12.2\dfrac{W}{m^2\cdot °C}$. <

Continued...

PROBLEM 1.56 (Cont.)

(b) Equation (1) may be solved iteratively to find T_2 for any emissivity ε. *IHT* was used for this purpose, yielding the following.



COMMENTS: (1) Note that as the wall emissivity increases, the surface temperature increases since the surroundings temperature is relatively hot. (2) The *IHT* code used in part (b) is shown below. (3) It is a good habit to work in temperature units of kelvins when radiation heat transfer is included in the solution of the problem.

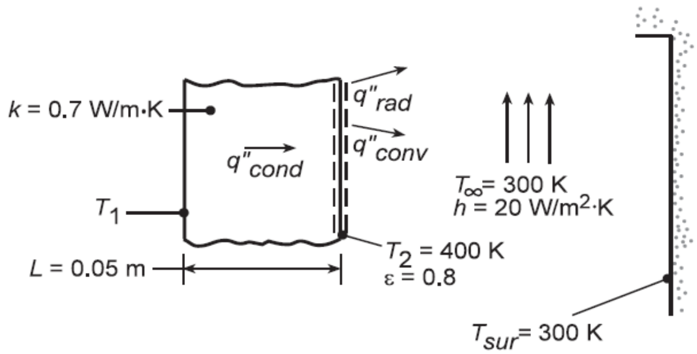
```
//Problem 1.56

h = 12.2  //W/m^2·K (convection coefficient)
L = 0.02  //m (sheet thickness)
k = 0.029 //W/m·K (thermal conductivity)
T1 = 30 + 273 //K (hot wall temperature)
Tsur = 320 //K (surroundings temperature)
sigma = 5.67*10^-8 //W/m^2·K^4 (Stefan -Boltzmann constant)
Tinf = 5 + 273 //K (ambient temperature)
e = 0.95 //emissivity

//Equation (1) is

k*(T1-T2)/L = h*(T2-Tinf) + e*sigma*(T2^4 - Tsur^4)
```

PROBLEM 1.57

KNOWN: Thickness and thermal conductivity, k, of an oven wall. Temperature and emissivity, ε, of front surface. Temperature and convection coefficient, h, of air. Temperature of large surroundings.

FIND: (a) Temperature of back surface, (b) Effect of variations in k, h and ε.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction, (3) Radiation exchange with large surroundings.

ANALYSIS: (a) Applying an energy balance, Eq. 1.13 to the front surface and substituting the appropriate rate equations, Eqs. 1.2, 1.3a and 1.7, find

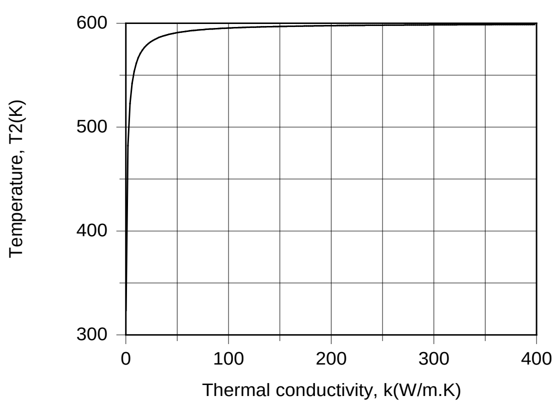
$$k\frac{T_1 - T_2}{L} = h\left(T_2 - T_\infty\right) + \varepsilon\sigma\left(T_2^4 - T_{sur}^4\right).$$

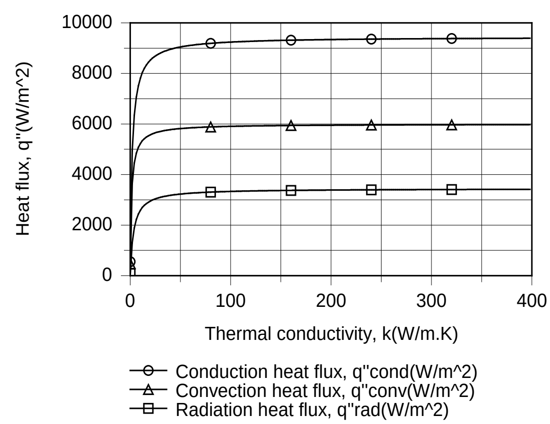
Substituting numerical values, find

$$T_1 - T_2 = \frac{0.05\,\text{m}}{0.7\,\text{W/m}\cdot\text{K}}\left[20\frac{\text{W}}{\text{m}^2\cdot\text{K}}100\,\text{K} + 0.8\times 5.67\times 10^{-8}\frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\left[\left(400\,\text{K}\right)^4 - \left(300\,\text{K}\right)^4\right]\right] = 200\,\text{K}.$$

Since T_2 = 400 K, it follows that T_1 = 600 K. <

(b) Parametric effects may be evaluated by using the IHT *First Law* Model for a *Nonisothermal Plane* Wall. Changes in k strongly influence conditions for k < 20 W/m·K, but have a negligible effect for larger values, as T_2 approaches T_1 and the heat fluxes approach the corresponding limiting values



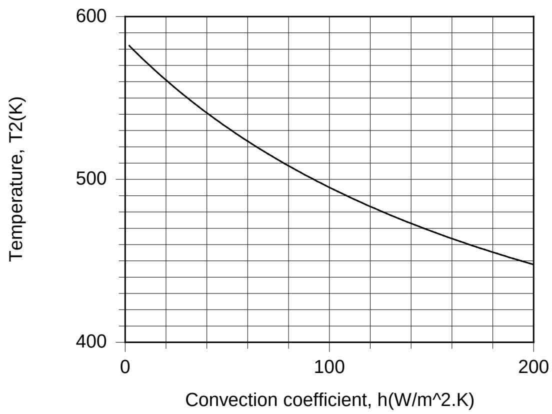


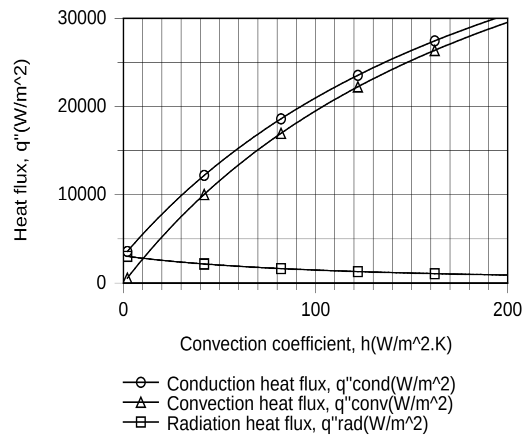
Continued…

PROBLEM 1.57 (Cont.)

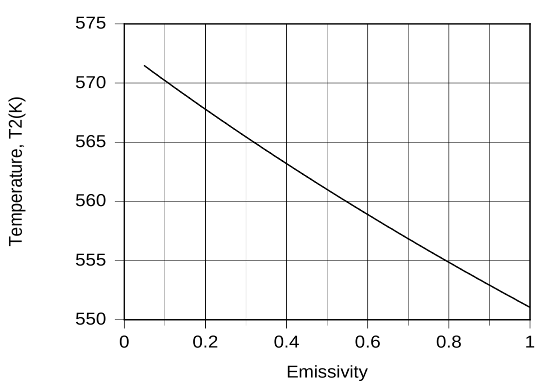
The implication is that, for k > 20 W/m·K, heat transfer by conduction in the wall is extremely efficient relative to heat transfer by convection and radiation, which become the *limiting* heat transfer processes. Larger fluxes could be obtained by increasing ε and h and/or by decreasing T_∞ and T_{sur}.

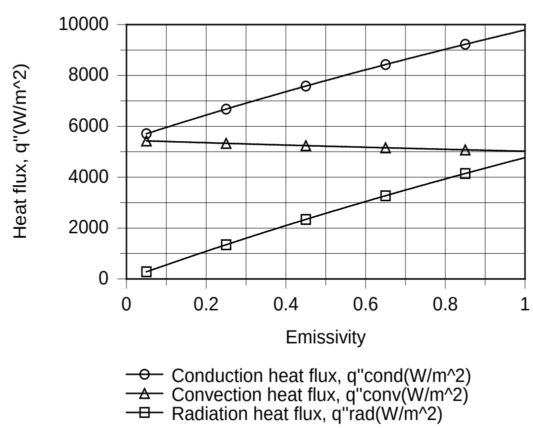
With increasing h, the front surface is cooled more effectively (T_2 decreases), and although q''_{rad} decreases, the reduction is exceeded by the increase in q''_{conv}. With a reduction in T_2 and fixed values of k and L, q''_{cond} must also increase.





The surface temperature also decreases with increasing ε, and the increase in q''_{rad} exceeds the reduction in q''_{conv}, allowing q''_{cond} to increase with ε.





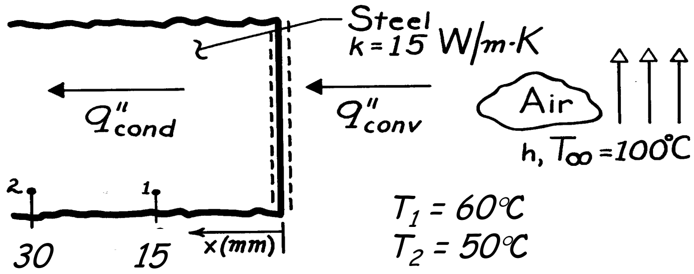
COMMENTS: Conservation of energy, of course, dictates that irrespective of the prescribed conditions, $q''_{cond} = q''_{conv} + q''_{rad}$.

PROBLEM 1.58

KNOWN: Temperatures at 15 mm and 30 mm from the surface and in the adjoining airflow for a thick stainless steel casting.

FIND: Surface convection coefficient, h.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) One-dimensional conduction in the x-direction, (3) Constant properties, (4) Negligible generation.

ANALYSIS: From a surface energy balance, it follows that

$$q''_{cond} = q''_{conv}$$

where the convection rate equation has the form

$$q''_{conv} = h\left(T_\infty - T_0\right),$$

and q''_{cond} can be evaluated from the temperatures prescribed at surfaces 1 and 2. That is, from Fourier's law,

$$q''_{cond} = k\frac{T_1 - T_2}{x_2 - x_1}$$

$$q''_{cond} = 15\frac{W}{m\cdot K}\frac{(60-50)^\circ C}{(30\text{-}15)\times 10^{-3}m} = 10,000\ W/m^2.$$

Since the temperature gradient in the solid must be linear for the prescribed conditions, it follows that

$$T_o = 70°C.$$

Hence, the convection coefficient is

$$h = \frac{q''_{cond}}{T_\infty - T_0}$$

$$h = \frac{10,000\ W/m^2}{30°C} = 333\ W/m^2\cdot K$$ <

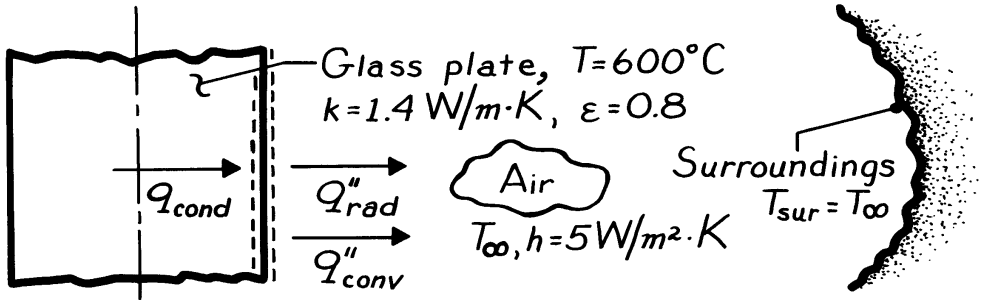
COMMENTS: The accuracy of this procedure for measuring h depends strongly on the validity of the assumed conditions.

PROBLEM 1.59

KNOWN: Conditions associated with surface cooling of plate glass which is initially at 600°C. Maximum allowable temperature gradient in the glass.

FIND: Lowest allowable air temperature, T_∞.

SCHEMATIC:



ASSUMPTIONS: (1) Surface of glass exchanges radiation with large surroundings at $T_{sur} = T_\infty$, (2) One-dimensional conduction in the x-direction.

ANALYSIS: The maximum temperature gradient will exist at the surface of the glass and at the instant that cooling is initiated. From the surface energy balance, Eq. 1.13, and the rate equations, Eqs. 1.1, 1.3a and 1.7, it follows that

$$-k\frac{dT}{dx} - h\left(T_s - T_\infty\right) - \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) = 0$$

or, with $(dT/dx)_{max}$ = -15°C/mm = -15,000°C/m and $T_{sur} = T_\infty$,

$$-1.4\frac{W}{m\cdot K}\left[-15,000\frac{\circ C}{m}\right] = 5\frac{W}{m^2\cdot K}\left(873 - T_\infty\right)K$$

$$+0.8\times 5.67\times 10^{-8}\frac{W}{m^2\cdot K^4}\left[873^4 - T_\infty^4\right]K^4.$$

T_∞ may be obtained from a trial-and-error solution, from which it follows that, for T_∞ = 618K,

$$21,000\frac{W}{m^2} \approx 1275\frac{W}{m^2} + 19,730\frac{W}{m^2}.$$

Hence the lowest allowable air temperature is

$$T_\infty \approx 618K = 345^\circ C. \qquad <$$

COMMENTS: (1) Initially, cooling is determined primarily by radiation effects.

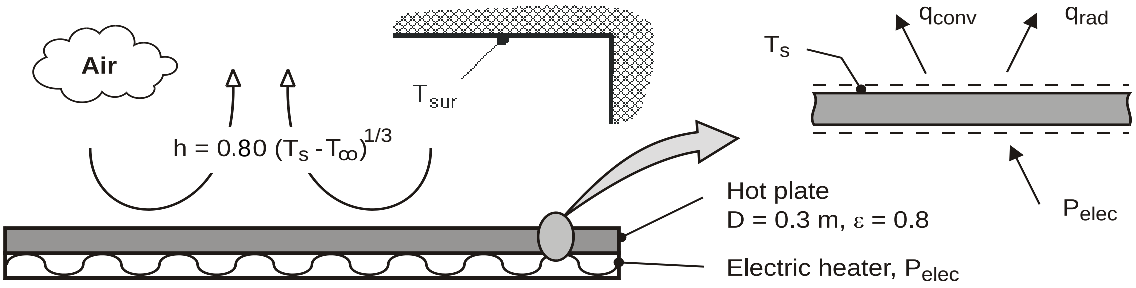
(2) For fixed T_∞, the surface *temperature gradient* would *decrease* with *increasing* time into the cooling process. Accordingly, T_∞ could be decreasing with increasing time and still keep within the maximum allowable temperature gradient.

PROBLEM 1.60

KNOWN: Surface temperature, diameter and emissivity of a hot plate. Temperature of surroundings and ambient air. Expression for convection coefficient.

FIND: (a) Operating power for prescribed surface temperature, (b) Effect of surface temperature on power requirement and on the relative contributions of radiation and convection to heat transfer from the surface.

SCHEMATIC:



ASSUMPTIONS: (1) Plate is of uniform surface temperature, (2) Walls of room are large relative to plate, (3) Negligible heat loss from bottom or sides of plate.

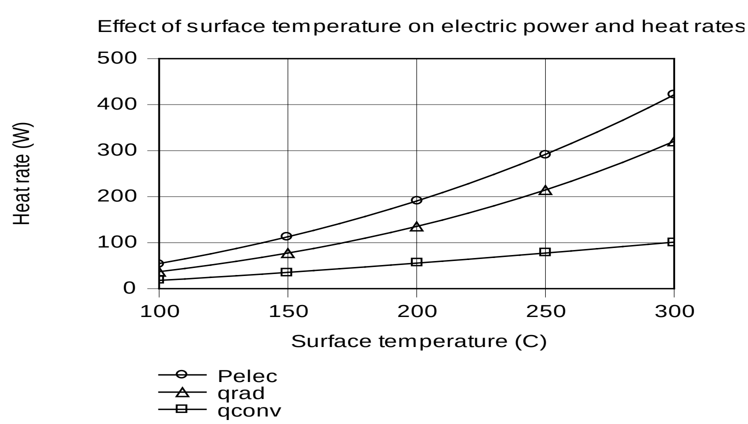
ANALYSIS: (a) From an energy balance on the hot plate, $P_{elec} = q_{conv} + q_{rad} = A_p$ $\left(q''_{conv} + q''_{rad}\right)$. Substituting for the area of the plate and from Eqs. (1.3a) and (1.7), with h = 0.80 $(T_s - T_\infty)^{1/3}$, it follows that

$$P_{elec} = \left(\pi D^2/4\right)\left[0.80\left(T_s - T_\infty\right)^{4/3} + \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)\right]$$

$$P_{elec} = \pi\left(0.3\text{m}\right)^2/4\left[0.80\left(175\right)^{4/3} + 0.8\times5.67\times10^{-8}\left(473^4 - 298^4\right)\right]\ \text{W/m}^2$$

$$P_{elec} = 0.0707\ \text{m}^2\left[783\ \text{W/m}^2 + 1913\ \text{W/m}^2\right] = 55.4\ \text{W} + 135.2\ \text{W} = 190.6\ \text{W}$$ <

(b) As shown graphically, both the radiation and convection heat rates, and hence the requisite electric power, increase with increasing surface temperature.



However, because of its dependence on the fourth power of the surface temperature, the increase in radiation is more pronounced. The significant relative effect of radiation is due to the small convection coefficients characteristic of natural convection, with $3.37 \le h \le 5.2$ W/m^2·K for $100 \le T_s < 300$°C.

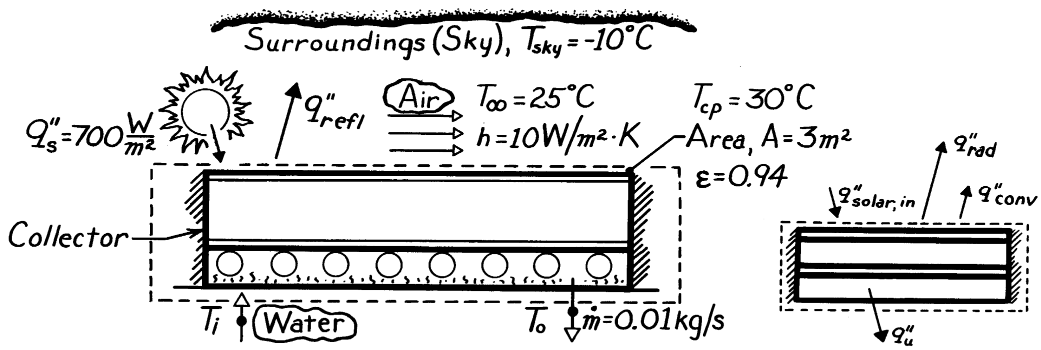
COMMENTS: Radiation losses could be reduced by applying a low emissivity coating to the surface, which would have to maintain its integrity over the range of operating temperatures.

PROBLEM 1.61

KNOWN: Solar collector designed to heat water operating under prescribed solar irradiation and loss conditions.

FIND: (a) Useful heat collected per unit area of the collector, q''_u, (b) Temperature rise of the water flow, $T_o - T_i$, and (c) Collector efficiency.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) No heat losses out sides or back of collector, (3) Collector area is small compared to sky surroundings.

PROPERTIES: *Table A.6*, Water (300K): c_p = 4179 J/kg·K.

ANALYSIS: (a) Defining the collector as the control volume and writing the conservation of energy requirement on a per unit area basis, find that

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \dot{E}_{st}.$$

Identifying processes as per above right sketch,

$$q''_{solar} - q''_{rad} - q''_{conv} - q''_u = 0$$

where $q''_{solar} = 0.9\ q''_S$; that is, 90% of the solar flux is absorbed in the collector (Eq. 1.6). Using the appropriate rate equations, the useful heat rate per unit area is

$$q''_u = 0.9\ q''_S - \varepsilon\sigma\left(T^4_{cp} - T^4_{sky}\right) - h\left(T_s - T_\infty\right)$$

$$q''_u = 0.9\times 700\frac{W}{m^2} - 0.94\times 5.67\times 10^{-8}\frac{W}{m^2\cdot K^4}\left(303^4 - 263^4\right)K^4 - 10\frac{W}{m^2\cdot K}(30-25)^\circ C$$

$$q''_u = 630\ W/m^2 - 194\ W/m^2 - 50\ W/m^2 = 386\ W/m^2.$$ <

(b) The total useful heat collected is $q''_u \cdot A$. Defining a control volume about the water tubing, the useful heat causes an enthalpy change of the flowing water. That is,

$$q''_u \cdot A = \dot{m}c_p\left(T_i - T_o\right) \qquad \text{or}$$

$$\left(T_i - T_o\right) = 386\ W/m^2 \times 3m^2 / 0.01kg/s \times 4179 J/kg\cdot K = 27.7^\circ C.$$ <

(c) The efficiency is $\eta = q''_u / q''_S = \left(386\ W/m^2\right)/\left(700\ W/m^2\right) = 0.55$ or 55%. <

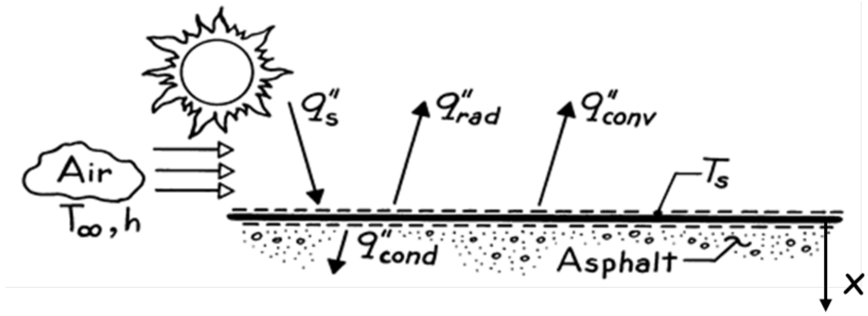
COMMENTS: Note how the sky has been treated as large surroundings at a uniform temperature T_{sky}.

PROBLEM 1.62(a)

KNOWN: Solar radiation is incident on an asphalt paving.

FIND: Relevant heat transfer processes.

SCHEMATIC:



The relevant processes shown on the schematic include:

q''_S Incident solar radiation flux, a large portion of which $q''_{S,abs}$, is absorbed by the asphalt surface,

q''_{rad} Net radiation from the surface,

q''_{conv} Convection heat transfer from the surface to the air, and

q''_{cond} Conduction heat transfer from the surface into the asphalt.

Applying the surface energy balance, Eq. 1.13,

$$q''_{S,abs} - q''_{rad} - q''_{conv} = q''_{cond}.$$

COMMENTS: (1) q''_{cond} and q''_{conv} could be evaluated from Eqs. 1.1 and 1.3, respectively.

(2) It has been assumed that the pavement surface temperature is higher than that of the underlying pavement and the air, in which case heat transfer by conduction and convection are from the surface.

(3) For simplicity, radiation incident on the pavement due to atmospheric emission has been ignored (see Section 12.8 for a discussion).

(4) Assuming irradiation from the air to the surface is negligible, the energy balance becomes

$$q''_{S,abs} - \varepsilon \sigma T_s^4 - h(T_s - T_\infty) = -k \left.\frac{dT}{dx}\right]_{x=0}.$$

PROBLEM 1.62(b)

KNOWN: Physical mechanism for microwave heating.

FIND: Comparison of (i) cooking in a microwave oven with a conventional radiant or convection oven and (ii) a microwave clothes dryer with a conventional dryer.

(i) Microwave cooking of food that contains water molecules occurs as a result of volumetric thermal energy generation *throughout* the food, without heating of the food container or the oven wall. Conventional cooking relies on radiant heat transfer from the oven walls and/or convection heat transfer from the air space to the surface of the food and subsequent heat transfer by conduction to the core of the food. Microwave cooking can be achieved in less time.

Heat loss from a microwave oven will likely be less than from a conventional oven. However, microwave cooking requires an electrical source of energy while the conventional oven can use, for example, natural gas. If the electricity for the microwave oven is generated by burning natural gas in a large power plant, the conventional natural gas oven will be more efficient than the microwave oven when the relatively low efficiency of the power plant is taken into account.

(ii) In a microwave dryer, the microwave radiation would heat the water, but not the fabric, directly (the fabric would be heated indirectly by thermal energy transfer from the water). By heating the water directly, energy would go directly into evaporation, unlike a conventional dryer where the walls and air are first heated electrically or by a gas heater, and thermal energy is subsequently transferred to the wet clothes. The microwave dryer would still require a rotating drum and air flow to remove the water vapor, but is able to operate more efficiently and at lower temperatures if the comparison is made to a conventional electric dryer.

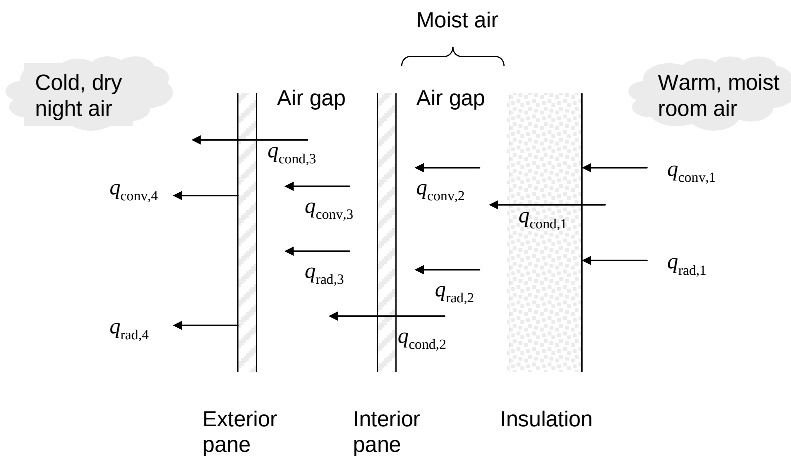
If comparison is made between the microwave dryer and a conventional dryer fueled by, for example, natural gas, the conventional dryer can be more efficient than the microwave dryer when the relatively low efficiency of the power plant is taken into account.

PROBLEM 1.62(c)

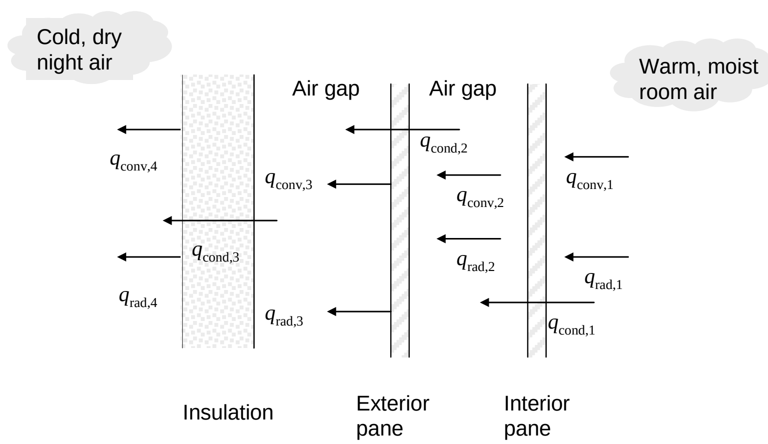
KNOWN: Double-pane windows with foamed insulation inside or outside. Cold, dry air outside and warm, moist air inside.

FIND: Identify heat transfer processes. Which configuration is preferred to avoid condensation?

SCHEMATIC:



Insulation on inside of window.



I

Insulation on outside of window.

ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer through window and insulation.

ANALYSIS: With the insulation on the inside, heat is transferred from the warm room air to the insulation by convection ($q_{conv,1}$) and from the warm interior surfaces of the room by radiation ($q_{rad,1}$). Heat is then conducted through the insulation ($q_{cond,1}$). From there, heat is transferred across the air gap between the insulation and the window by free convection ($q_{conv,2}$) and radiation ($q_{rad,2}$). Heat is transferred through the first glass pane by conduction ($q_{cond,2}$). Heat transfer across the air gap between the window panes occurs by free convection ($q_{conv,3}$) and radiation ($q_{rad,3}$). Heat is then transferred through the second glass pane by conduction ($q_{cond,3}$). From there, heat is transferred to the cold air by convection ($q_{conv,4}$) and to the cold surroundings by radiation ($q_{rad,4}$). The same mechanisms occur with the insulation on the outside of the window, just in a different order.

Continued...

Condensation may occur on any surface that is exposed to moist air if the surface temperature is below the dewpoint temperature. Condensation causes an additional heat transfer mechanism because when water vapor condenses it releases the enthalpy of vaporization ($q_{condense}$), which heats the surface on which condensation is occurring. For example, if condensation occurs on the inside surface of the window, this will increase the temperature of that surface and the rate of heat transfer through that window pane. The condensation heat transfer processes are not shown on the schematics.

We know that condensation does not occur on the window's interior pane when there is no insulation in place. If insulation were to be placed on the outside of the window, it will increase the temperature difference between the outside air and the window panes, increasing the window pane temperatures. Therefore condensation will still not occur. <

If the insulation is placed on the inside of the window, it will increase the temperature difference between the warm room air and the window's interior pane. Both the inner surface of the window and the side of the insulation facing the window may experience temperatures below the dewpoint temperature. Because the insulation is loosely-fitting, moist air infiltrates the gap between the insulation and the inner window pane, and condensation may occur. Liquid water may accumulate and cause water damage. To avoid condensation and associated water damage, the insulation should be placed on the outside of the window. <

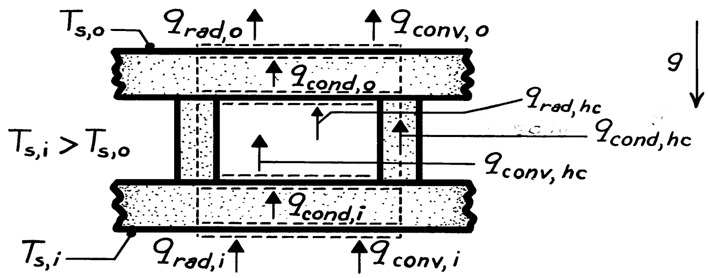
COMMENTS: (1) The potential water damage is *not* caused by window leakage. Any condensation problem would be exacerbated by adding more insulation to the inside of the window. (2) The potential for condensation damage would be reduced by lowering the humidity in the room, at the risk of increasing discomfort and the potential for illness. (3) Moisture may infiltrate through the insulation. Even tightly-fitting, improperly placed insulation can lead to condensation and water damage. (4) Adding the insulation to the exterior of the window will reduce the possibility of water damage due to condensation, but it cannot be easily removed to enjoy a bright winter day.

PROBLEM 1.62(d)

KNOWN: Geometry of a composite insulation consisting of a honeycomb core.

FIND: Relevant heat transfer processes.

SCHEMATIC:



The above schematic represents the cross section of a single honeycomb cell and surface slabs. Assumed direction of gravity field is downward. Assuming that the bottom (inner) surface temperature exceeds the top (outer) surface temperature $\left(T_{s,i} > T_{s,o}\right)$, heat transfer is in the direction shown.

Heat may be transferred to the inner surface by convection and radiation, whereupon it is transferred through the composite by

$q_{cond,i}$	Conduction through the inner solid slab,
$q_{conv,hc}$	Free convection through the cellular airspace,
$q_{cond,hc}$	Conduction through the honeycomb wall,
$q_{rad,hc}$	Radiation between the honeycomb surfaces, and
$q_{cond,o}$	Conduction through the outer solid slab.

Heat may then be transferred from the outer surface by convection and radiation. Note that for a single cell under steady state conditions,

$$q_{rad,i} + q_{conv,i} = q_{cond,i} = q_{conv,hc} + q_{cond,hc}$$

$$+ q_{rad,hc} = q_{cond,o} = q_{rad,o} + q_{conv,o}.$$

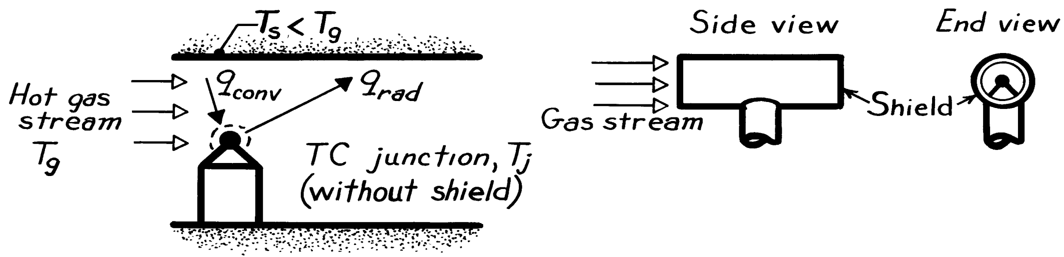
COMMENTS: Performance would be enhanced by using materials of low thermal conductivity, k, and emissivity, ε. Evacuating the airspace would enhance performance by eliminating heat transfer due to free convection.

PROBLEM 1.62(e)

KNOWN: A thermocouple junction is used, with or without a radiation shield, to measure the temperature of a gas flowing through a channel. The wall of the channel is at a temperature much less than that of the gas.

FIND: (a) Relevant heat transfer processes, (b) Temperature of junction relative to that of gas, (c) Effect of radiation shield.

SCHEMATIC:



ASSUMPTIONS: (1) Junction is small relative to channel walls, (2) Steady-state conditions, (3) Negligible heat transfer by conduction through the thermocouple leads.

ANALYSIS: (a) The relevant heat transfer processes are:

q_{rad}	Net radiation transfer from the junction to the walls, and
q_{conv}	Convection transfer from the gas to the junction.

(b) From a surface energy balance on the junction,

$$q_{conv} = q_{rad}$$

or from Eqs. 1.3a and 1.7,

$$h\,A\left(T_g - T_j\right) = \varepsilon\,A\,\sigma\left(T_j^4 - T_s^4\right).$$

To satisfy this equality, it follows that

$$T_s < T_j < T_g.$$

That is, the junction assumes a temperature between that of the channel wall and the gas, thereby sensing a temperature which is less than that of the gas.

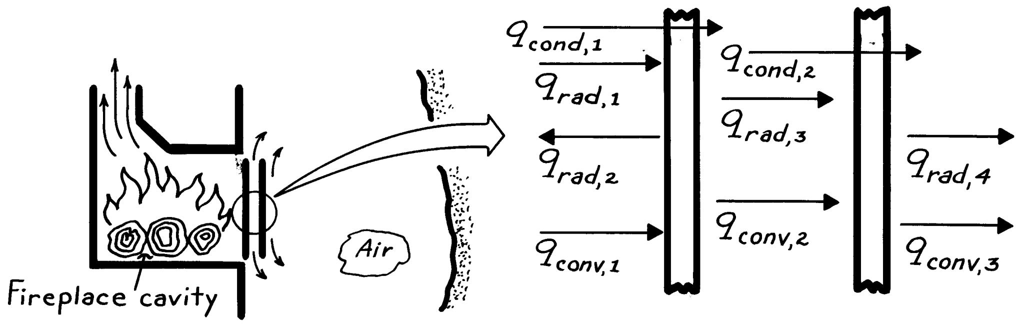
(c) The measurement error $\left(T_g - T_j\right)$ is reduced by using a radiation shield as shown in the schematic. The junction now exchanges radiation with the shield, whose temperature must exceed that of the channel wall. The radiation loss from the junction is therefore reduced, and its temperature more closely approaches that of the gas.

PROBLEM 1.62(f)

KNOWN: Fireplace cavity is separated from room air by two glass plates, open at both ends.

FIND: Relevant heat transfer processes.

SCHEMATIC:



The relevant heat transfer processes associated with the double-glazed, glass fire screen are:

$q_{rad,1}$	Radiation from flames and cavity wall, portions of which are absorbed and transmitted by the two panes,
$q_{rad,2}$	Emission from inner surface of inner pane to cavity,
$q_{rad,3}$	Net radiation exchange between outer surface of inner pane and inner surface of outer pane,
$q_{rad,4}$	Net radiation exchange between outer surface of outer pane and walls of room,
$q_{conv,1}$	Convection between cavity gases and inner pane,
q_{conv2}	Convection across air space between panes,
$q_{conv,3}$	Convection from outer surface to room air,
$q_{cond,1}$	Conduction across inner pane, and
$q_{cond,2}$	Conduction across outer pane.

COMMENTS: (1) Much of the luminous portion of the flame radiation is transmitted to the room interior.

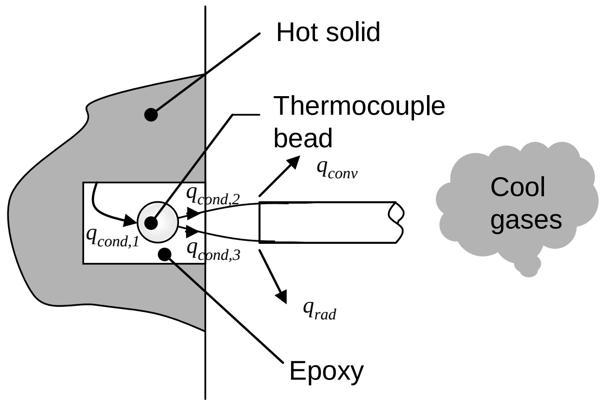
(2) All convection processes are buoyancy driven (free convection).

PROBLEM 1.62(g)

KNOWN: Thermocouple junction held in small hole in solid material by epoxy. Solid is hotter than surroundings.

FIND: Identify heat transfer processes. Will thermocouple junction sense temperature less than, equal to, or greater than solid temperature? How will thermal conductivity of epoxy affect junction temperature?

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions.

ANALYSIS: Heat is transferred from the solid material through the epoxy to the thermocouple junction by conduction, $q_{\text{cond,1}}$. Heat is also transferred from the junction along the thermocouple wires and their sheathing by conduction ($q_{\text{cond,2}}$ and $q_{\text{cond,3}}$), and from there to the surroundings by convection (q_{conv}) and radiation (q_{rad}). Thus, the junction is heated by the solid and cooled by the surroundings, and its temperature will be between the solid temperature and the temperature of the cool gases.

The junction temperature will be less than the solid temperature. <

Under steady-state conditions, the rate at which heat is transferred to the junction from the solid material must equal the rate at which heat is transferred from the junction to the cool gases and surroundings. If we think of this heat transfer rate as fixed, then Equation 1.2 shows that a higher thermal conductivity for the epoxy will result in a smaller temperature difference across the epoxy. This leads to the thermocouple sensing a temperature that is closer to the solid temperature.

Higher thermal conductivity of epoxy leads to the thermocouple temperature being closer to the solid temperature. <

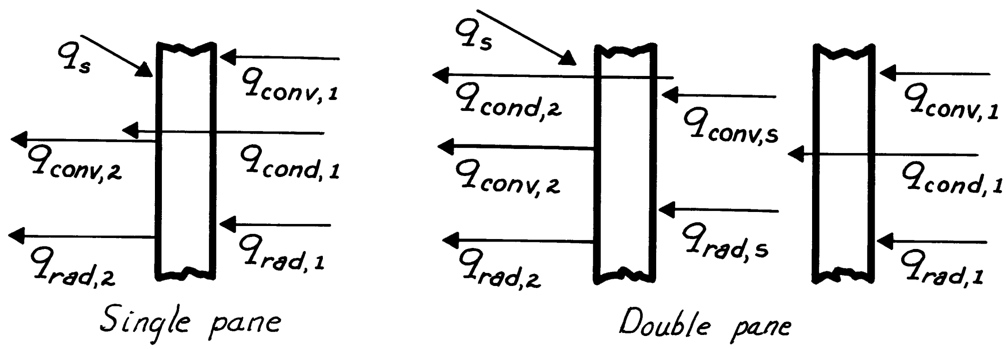
COMMENTS: (1) High thermal conductivity epoxies are formulated specifically for the purpose of affixing thermocouples. Their thermal conductivity is increased by adding small particles of high thermal conductivity materials such as silver. (2) Different types of thermocouple wires are available. To further reduce temperature differences between the solid and the thermocouple junction, small diameter thermocouple wires of relatively low thermal conductivity, such as chromel and alumel, are preferred. (3) Because thermocouple wires are made of different metals, in general $q_{\text{cond,2}} \neq q_{\text{cond,3}}$.

PROBLEM 1.63(a)

KNOWN: Room air is separated from ambient air by one or two glass panes.

FIND: Relevant heat transfer processes.

SCHEMATIC:



The relevant processes associated with single (above left schematic) and double (above right schematic) glass panes include.

$q_{conv,1}$	Convection from room air to inner surface of first pane,
$q_{rad,1}$	Net radiation exchange between room walls and inner surface of first pane,
$q_{cond,1}$	Conduction through first pane,
$q_{conv,s}$	Convection across airspace between panes,
$q_{rad,s}$	Net radiation exchange between outer surface of first pane and inner surface of second pane (across airspace),
$q_{cond,2}$	Conduction through a second pane,
$q_{conv,2}$	Convection from outer surface of single (or second) pane to ambient air,
$q_{rad,2}$	Net radiation exchange between outer surface of single (or second) pane and surroundings such as the ground, and
q_S	Incident solar radiation during day; fraction transmitted to room is smaller for double pane.

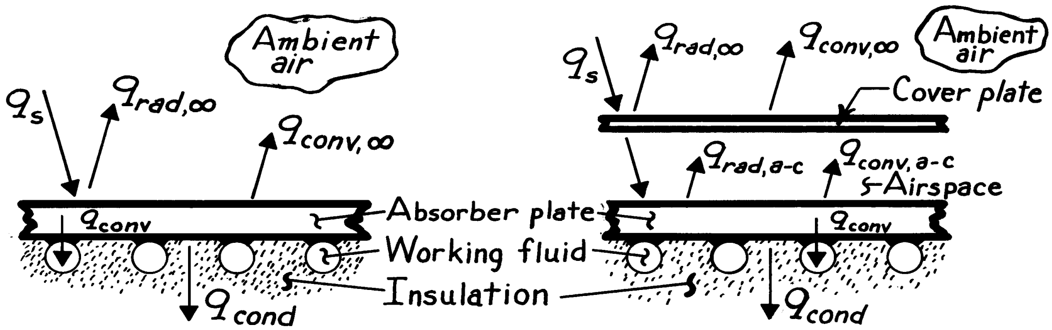
COMMENTS: Heat loss from the room is significantly reduced by the double pane construction.

PROBLEM 1.63(b)

KNOWN: Configuration of a flat plate solar collector.

FIND: Relevant heat transfer processes with and without a cover plate.

SCHEMATIC:



The relevant processes without (above left schematic) and with (above right schematic) include:

q_S	Incident solar radiation, a large portion of which is absorbed by the absorber plate. Reduced with use of cover plate (primarily due to reflection off cover plate).
$q_{rad,\infty}$	Net radiation exchange between absorber plate or cover plate and surroundings,
$q_{conv,\infty}$	Convection from absorber plate or cover plate to ambient air,
$q_{rad,a-c}$	Net radiation exchange between absorber and cover plates,
$q_{conv,a-c}$	Convection heat transfer across airspace between absorber and cover plates,
q_{cond}	Conduction through insulation, and
q_{conv}	Convection to working fluid.

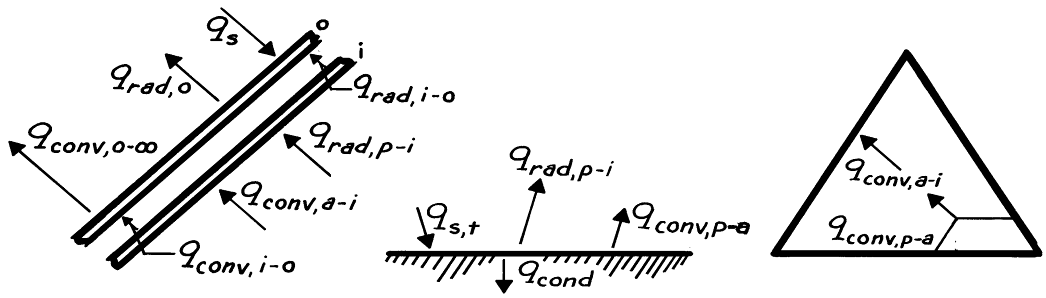
COMMENTS: The cover plate acts to significantly reduce heat losses by convection and radiation from the absorber plate to the surroundings.

PROBLEM 1.63(c)

KNOWN: Configuration of a solar collector used to heat air for agricultural applications.

FIND: Relevant heat transfer processes.

SCHEMATIC:



Assume the temperature of the absorber plates exceeds the ambient air temperature. At the *cover plates*, the relevant processes are:

$q_{conv,a-i}$	Convection from inside air to inner surface,
$q_{rad,p-i}$	Net radiation transfer from absorber plates to inner surface,
$q_{conv,i-o}$	Convection across airspace between covers,
$q_{rad,i-o}$	Net radiation transfer from inner to outer cover,
$q_{conv,o-\infty}$	Convection from outer cover to ambient air,
$q_{rad,o}$	Net radiation transfer from outer cover to surroundings, and
q_S	Incident solar radiation.

Additional processes relevant to the *absorber plates* and *airspace* are:

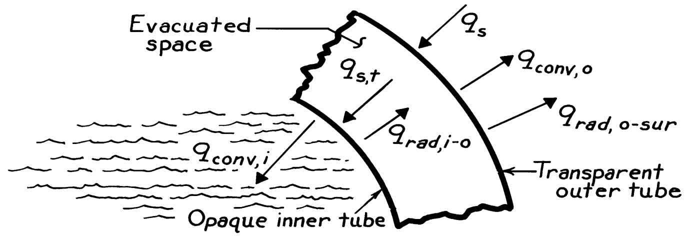
$q_{S,t}$	Solar radiation transmitted by cover plates,
$q_{conv,p-a}$	Convection from absorber plates to inside air, and
q_{cond}	Conduction through insulation.

PROBLEM 1.63(d)

KNOWN: Features of an evacuated tube solar collector.

FIND: Relevant heat transfer processes for one of the tubes.

SCHEMATIC:



The relevant heat transfer processes for one of the evacuated tube solar collectors includes:

q_S	Incident solar radiation including contribution due to reflection off panel (most is transmitted),
$q_{conv,o}$	Convection heat transfer from outer surface to ambient air,
$q_{rad,o-sur}$	Net rate of radiation heat exchange between outer surface of outer tube and the surroundings, including the panel,
$q_{S,t}$	Solar radiation transmitted through outer tube and incident on inner tube (most is absorbed),
$q_{rad,i-o}$	Net rate of radiation heat exchange between outer surface of inner tube and inner surface of outer tube, and
$q_{conv,i}$	Convection heat transfer to working fluid.

There is also conduction heat transfer through the inner and outer tube walls. If the walls are thin, the temperature drop across the walls will be small.